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OF SOUTH AFRICA

The Business of Change: 2010 and Beyond

Generating Interest Rate Scenarios for Fixed Income Portfolio Optimisation

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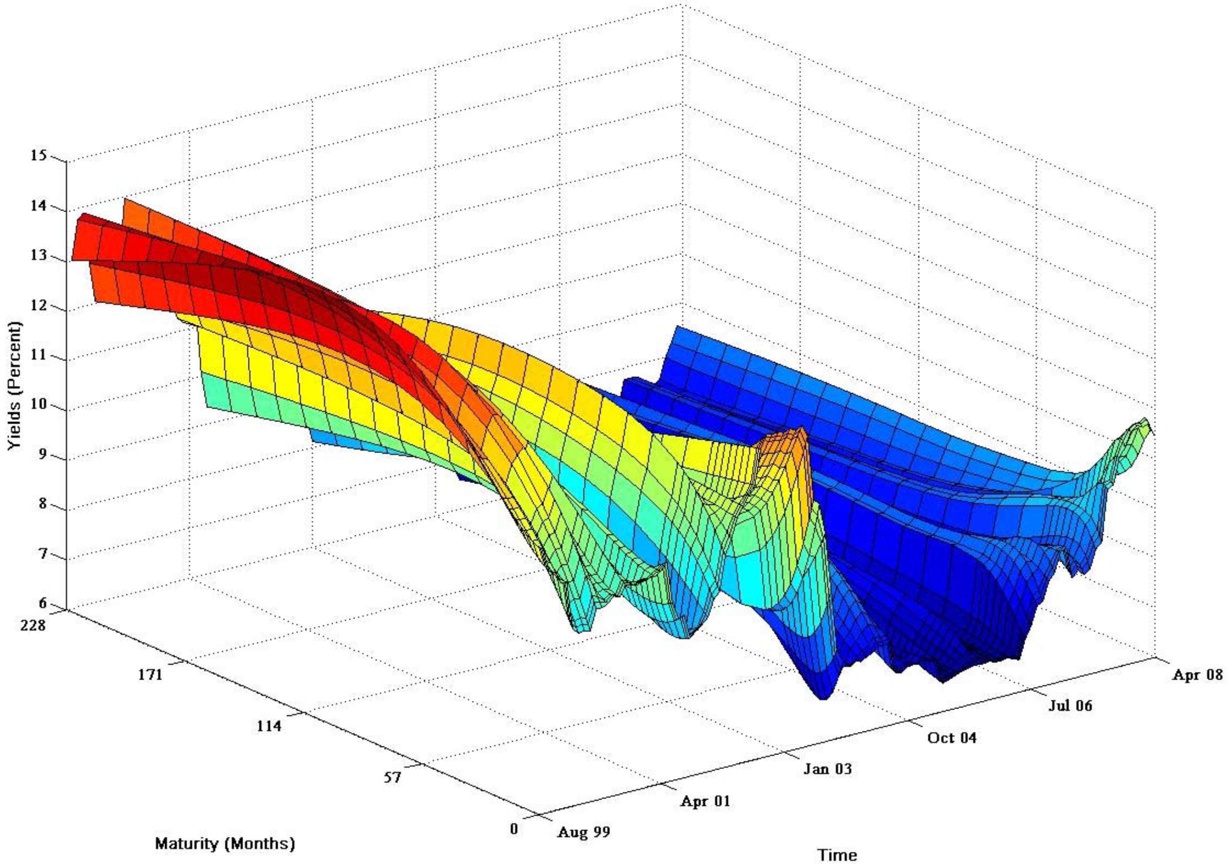
Agenda

- Background
- Yields-only model
- Yields-macro model
- Out-of-sample testing
- Scenario generation
- Back-testing
- Conclusion

Background

- One of the main sources of uncertainty in analysing risk and return properties of fixed income portfolios is the stochastic evolution of the shape of the term structure of interest rates
- Diebold et al. (2006) – The macro economy and the yield curve: a dynamic latent factor approach
  - three factor term structure model (Nelson-Siegel, 1987)
  - level, slope and curvature – Diebold & Li (2006)
  - macro-economic factors (real activity, inflation and monetary policy)
- Model South African term structure using a Kalman filter approach
- Four latent factors and macro-economic factors (capacity utilisation, inflation and repo-rate)

Background



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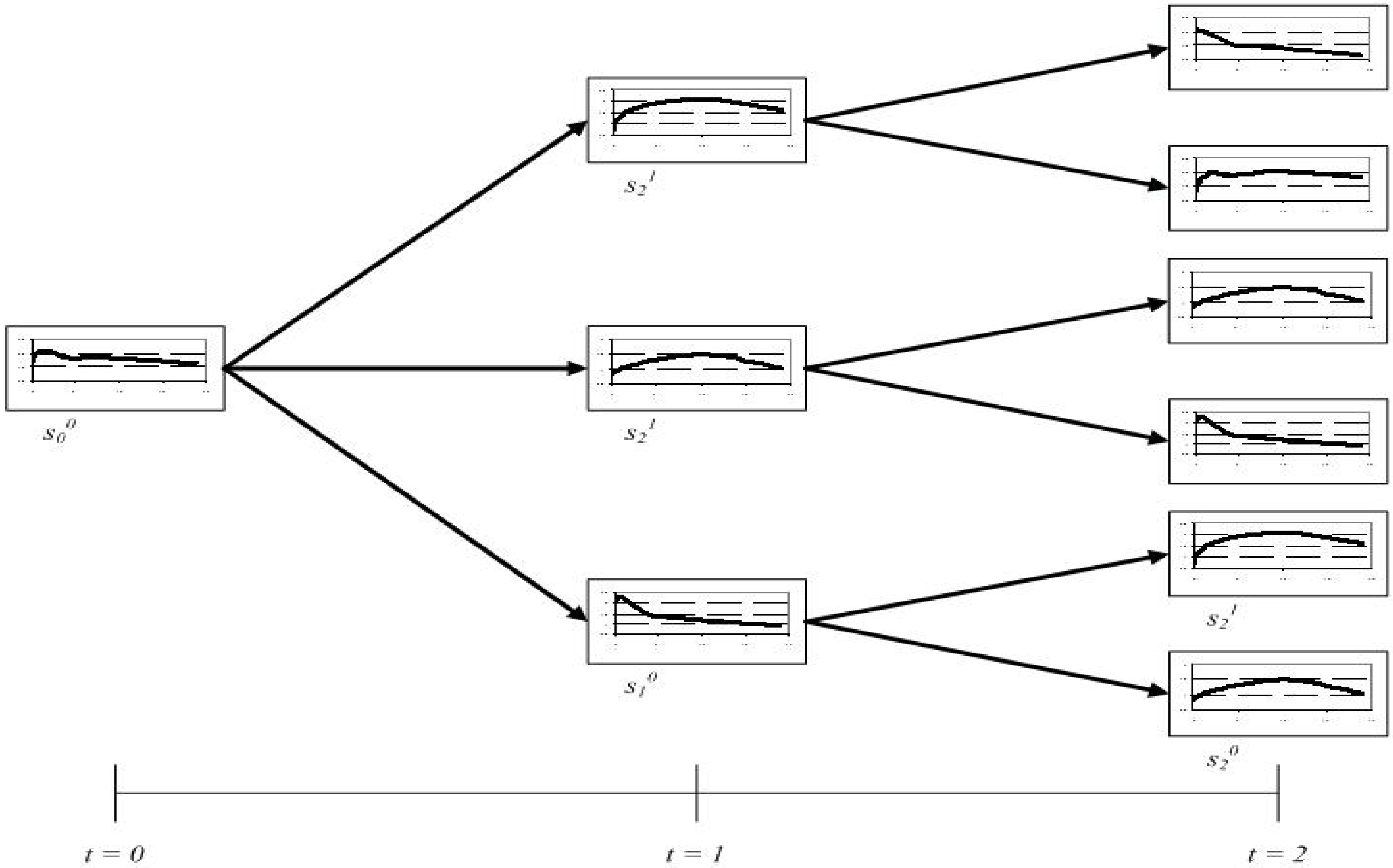
Background



Yields-only model

- Factor model approach
  - Represent larger set of yields as a function of a smaller set of unobservable factors
  - Nelson-Siegel (Nelson & Siegel, 1987)

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right)$$

  - Level, slope and curvature (Diebold & Li, 2002)

$$y_t(\tau) = L_t + S_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right)$$

Yields-only model

- Transition equation
  - modeled as VAR(1)

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}$$

- Measurement equation

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_3} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{pmatrix}$$

- Matrix Notation

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t$$

$$y_t = \Lambda f_t + \varepsilon_t \,.$$

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Yields-only model

- Linear least-squares optimality of the Kalman filter

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right]$$

$$E\left(f_0 \eta_t'\right) = 0\,,$$

$$E\left(f_0 \varepsilon_t'\right) = 0\,.$$

  - Q assumed to be non-diagonal and H assumed to be diagonal
- BEASSA – Perfect fit bond curve
- Maturities – 1, 2, 3, 6, 9, 12, 15, 18, 21, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192, 204, 216, 228 months
- End of month data form April 2000 through April 2008

Yields-only model

- Parameter estimation
  - transition matrix A (9 parameters)
  - mean state vector μ (3 parameters)
  - measurement matrix Λ (1 parameter)
  - covariance matrix Q (6 parameters)
  - covariance matrix H (27 parameters)
  - 46 Parameters
- Maximize Gaussian likelihood function
- SAS (Proc NLP and Proc IML)

Yields-only model

- Estimated *A* matrix

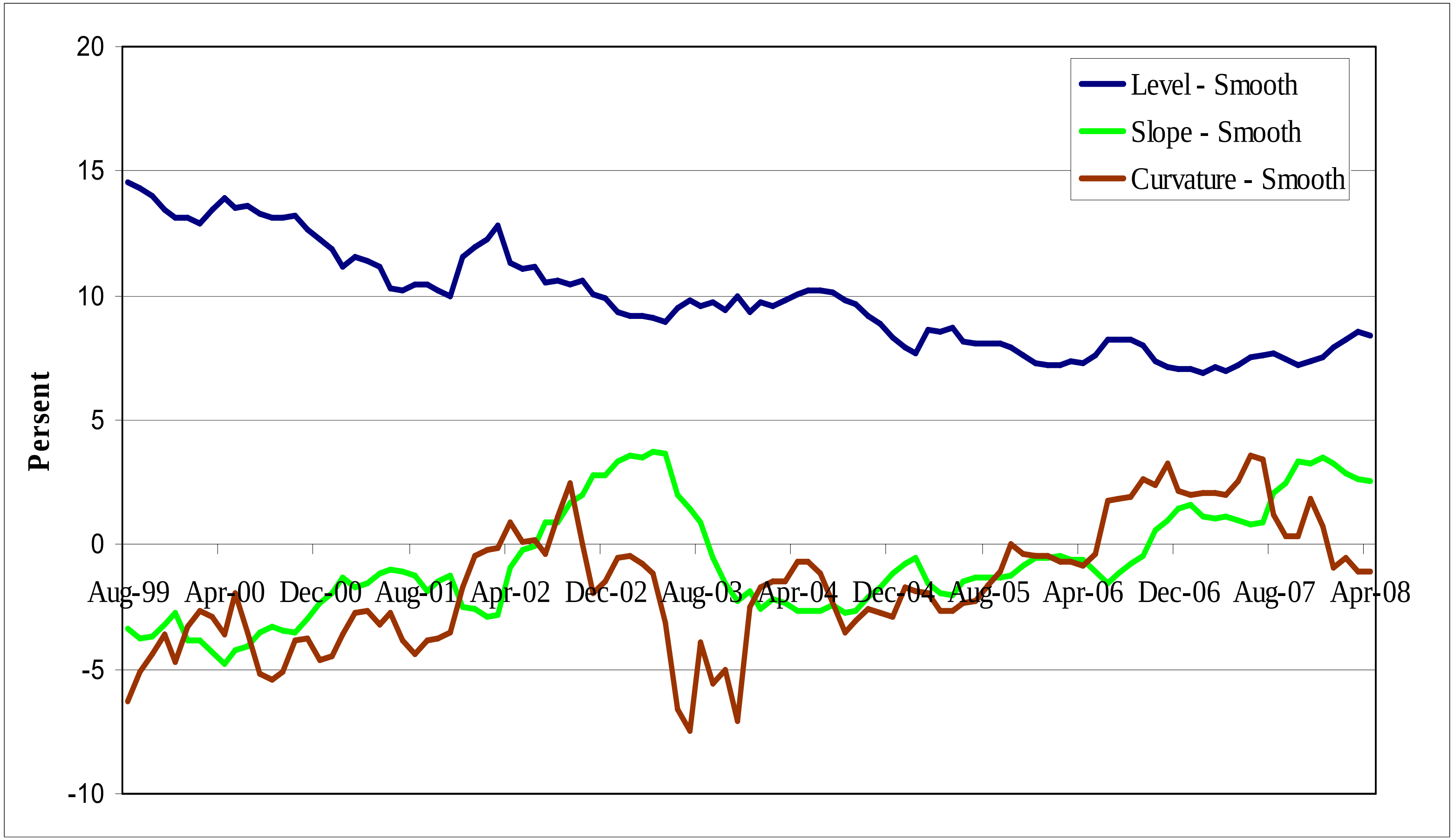
	L_{t-1}	S_{t-1}	C_{t-1}	μ
L_t	**0.956** (0.026)	0.004 (0.022)	-0.010 (0.019)	**8.353** (1.154)
S_t	**0.085** (0.031)	**0.958** (0.027)	**0.129** (0.023)	0.383 (1.153)
C_t	**-0.154** (0.073)	-0.120 (0.063)	**0.856** (0.053)	-0.701 (0.758)

- Estimated *Q* matrix

	L_t	S_t	C_t
L_t	**0.143** (0.020)	**-0.136** (0.022)	-0.046 (0.040)
S_t		**0.214** (0.030)	0.061 (0.049)
C_t			**1.136** (0.158)

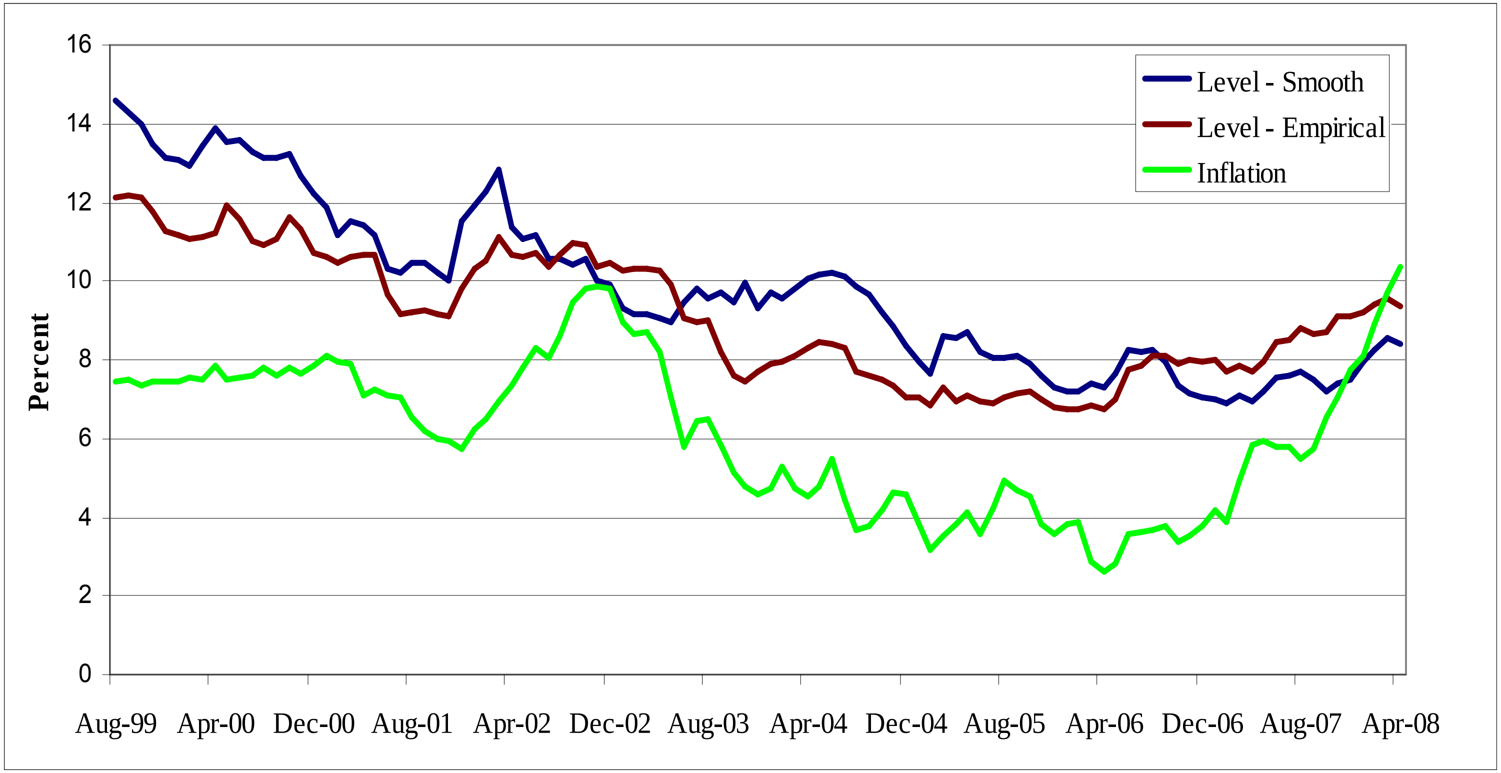
Yields-only model

- Level, slope and curvature



Yields-only model

- Level [Empirical level = $(y_3 + y_{24} + y_{228})/3$]



	Level - Smooth	*Level - Empirical*	*Inflation*
Level - Smooth	1		
Level - Empirical	0.839483157	1	
Inflation	0.543439854	0.833657289	1

Yields-only model

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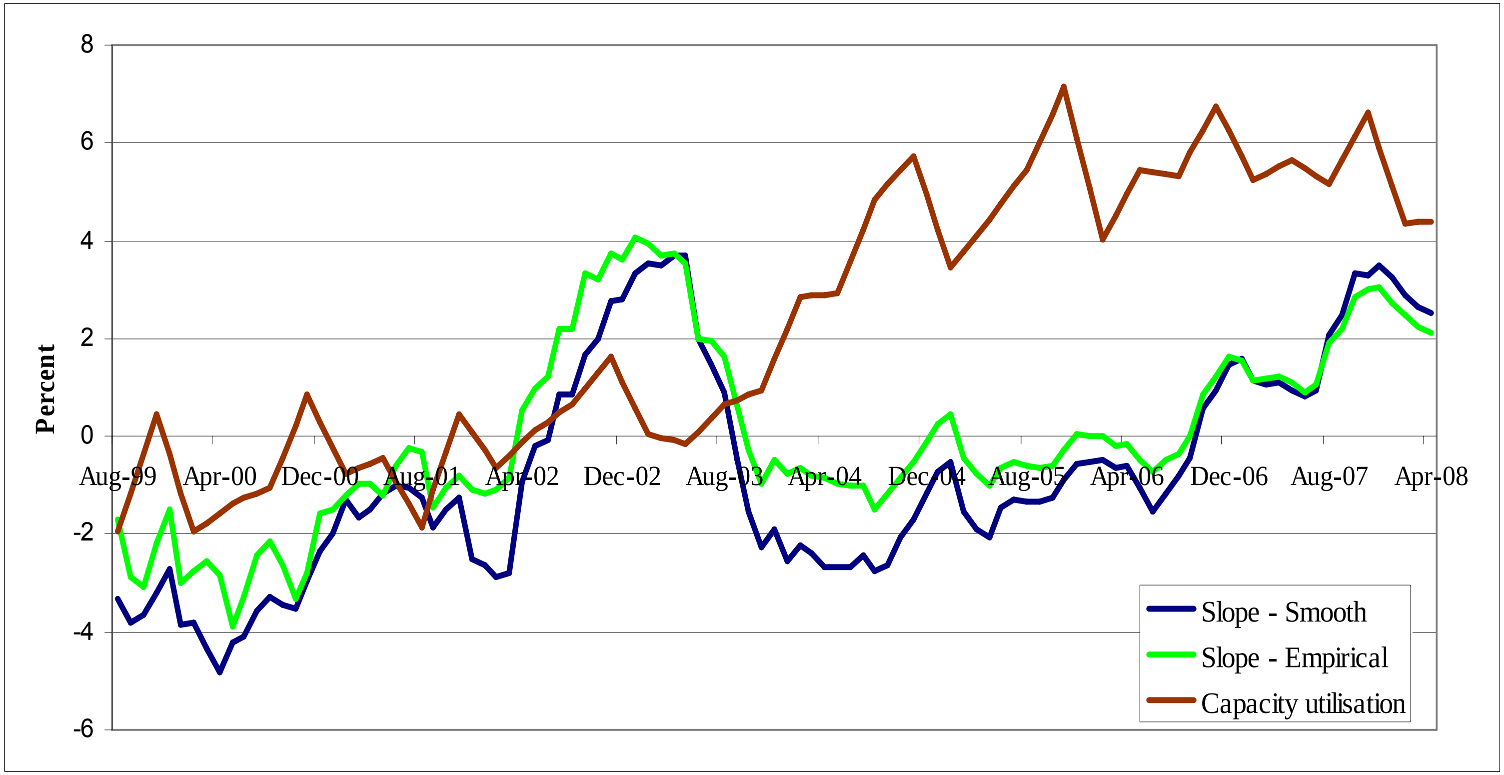
- Slope [Empirical slope = (y_3 - y_{228})]



	Slope - Smooth	*Slope - Empirical*	*Capacity utilisation*
Slope - Smooth	1		
Slope - Empirical	0.967509355	1	
Capacity utilisation	0.365349447	0.312969954	1

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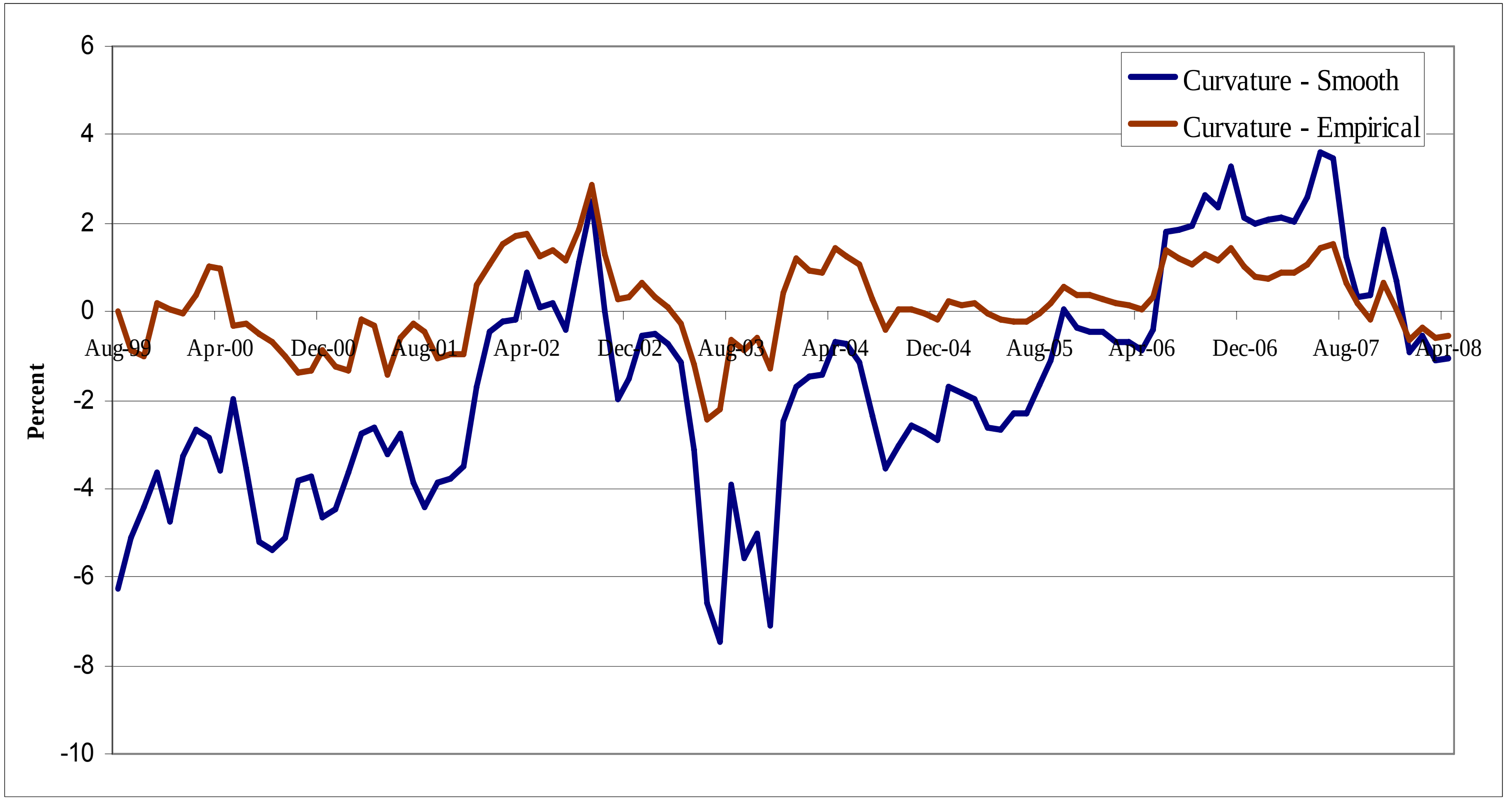
Yields-only model

- Curvature [Empirical curvature = $(2y_{24} - y_3 - y_{228})$]



	Curvature - Smooth	*Curvature - Empirical*
Curvature - Smooth	1	
Curvature - Empirical	0.781420326	1

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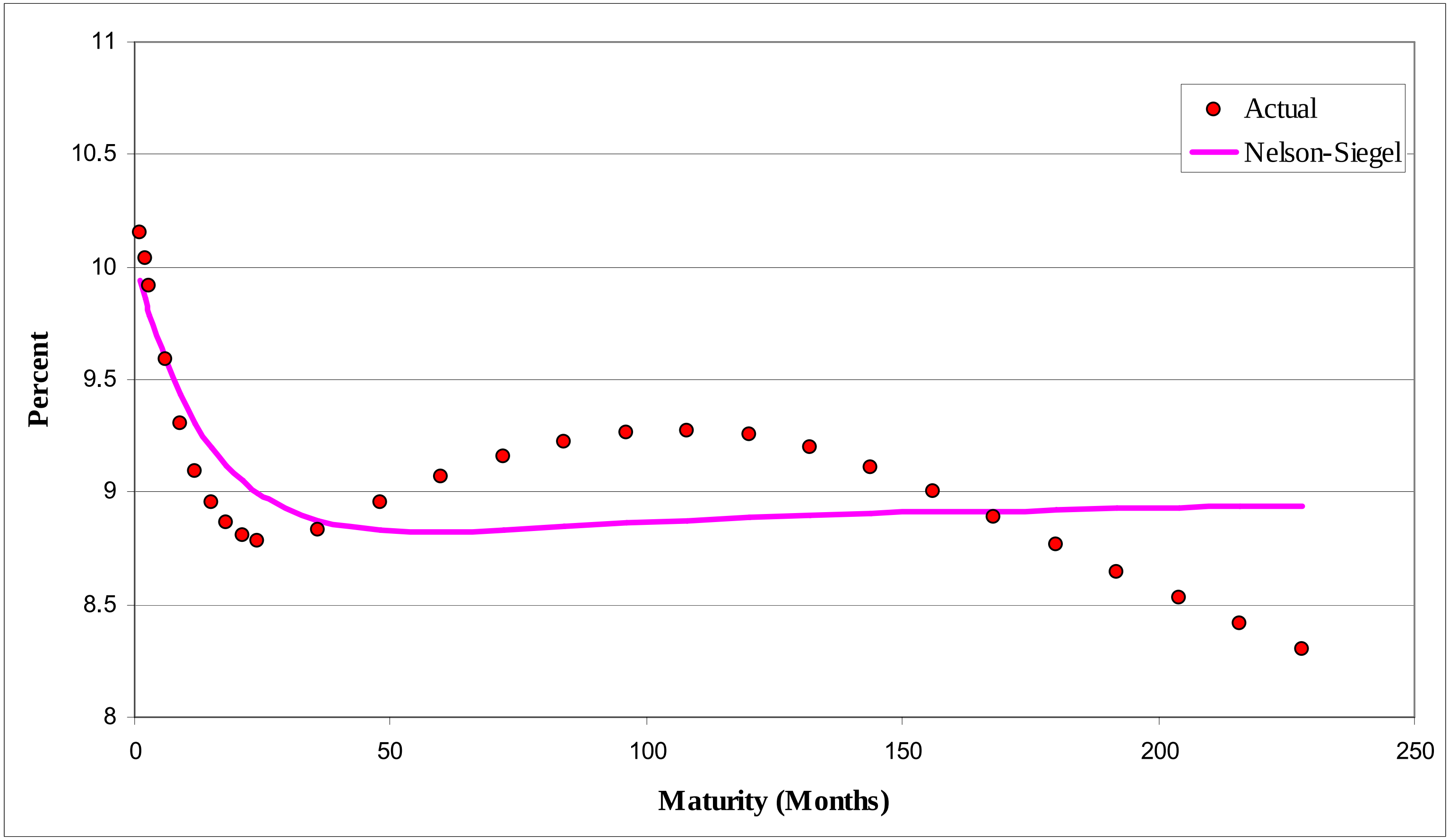
Yields-only model

- Measurement errors

Maturity	Yields-only				Yields-macro			
	3 Factor		4 Factor		3 Factor		4 Factor	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	-0.0265	0.3633	-0.0147	0.4762	0.00499	0.45207	-0.00356	0.41848
2	0.0002	0.3484	0.0023	0.4578	0.00817	0.44751	-0.01295	0.40813
3	0.0190	0.3460	0.0139	0.4544	0.00619	0.32912	0.03356	0.32738
6	0.0406	0.3596	0.0246	0.4736	-0.02549	0.35777	0.04737	0.33934
9	0.0372	0.3793	0.0209	0.5000	0.00047	0.34028	0.04164	0.33493
12	0.0298	0.3996	0.0188	0.5231	0.01876	0.33617	0.03652	0.33680
15	0.0248	0.4173	0.0212	0.5414	0.03973	0.34937	0.02497	0.35622
18	0.0205	0.4332	0.0240	0.5564	0.03634	0.36995	0.01856	0.38057
21	0.0171	0.4475	0.0264	0.5691	0.02913	0.38991	0.01649	0.40204
24	0.0138	0.4602	0.0267	0.5796	0.02419	0.40648	0.01736	0.41974
36	0.0066	0.4992	0.0115	0.6069	0.01982	0.42086	0.01931	0.43520
48	0.0144	0.5202	-0.0136	0.6194	0.01649	0.43349	0.02158	0.44886
60	0.0308	0.5285	-0.0352	0.6254	0.01308	0.44459	0.02343	0.46086
72	0.0472	0.5309	-0.0483	0.6288	0.00570	0.47664	0.02375	0.49547
84	0.0577	0.5323	-0.0521	0.6305	0.01297	0.49193	0.01631	0.51160
96	0.0585	0.5350	-0.0487	0.6310	0.02864	0.49666	0.00857	0.51633
108	0.0465	0.5398	-0.0417	0.6311	0.04441	0.49728	0.00546	0.51626
120	0.0202	0.5469	-0.0346	0.6317	0.05450	0.49787	0.00824	0.51527
132	-0.0202	0.5571	-0.0296	0.6330	0.05513	0.50052	0.01532	0.51502
144	-0.0733	0.5716	-0.0276	0.6349	0.04336	0.50604	0.02373	0.51603
156	-0.1358	0.5910	-0.0277	0.6367	0.01775	0.51479	0.03088	0.51807
168	-0.2038	0.6157	-0.0277	0.6383	-0.02155	0.52752	0.03526	0.52059
180	-0.2746	0.6454	-0.0268	0.6396	-0.07321	0.54507	0.03660	0.52321
192	-0.3467	0.6794	-0.0248	0.6408	-0.13401	0.56809	0.03615	0.52592
204	-0.4189	0.7169	-0.0217	0.6419	-0.20009	0.59659	0.03541	0.52861
216	-0.4900	0.7572	-0.0173	0.6431	-0.26898	0.62999	0.03514	0.53130
228	-0.5591	0.7997	-0.0112	0.6450	-0.33908	0.66765	0.03560	0.53413

Yields-only model



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Yields-only model

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- Four factor model
  - Svensson (Svensson, 1994)

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right) + \beta_3 \left(\frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right) + \beta_4 \left(\frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right)$$

  - factor representation

$$y_t(\tau) = F_{1t} + F_{2t} \left(\frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right) + F_{3t} \left(\frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right) + F_{4t} \left(\frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right)$$

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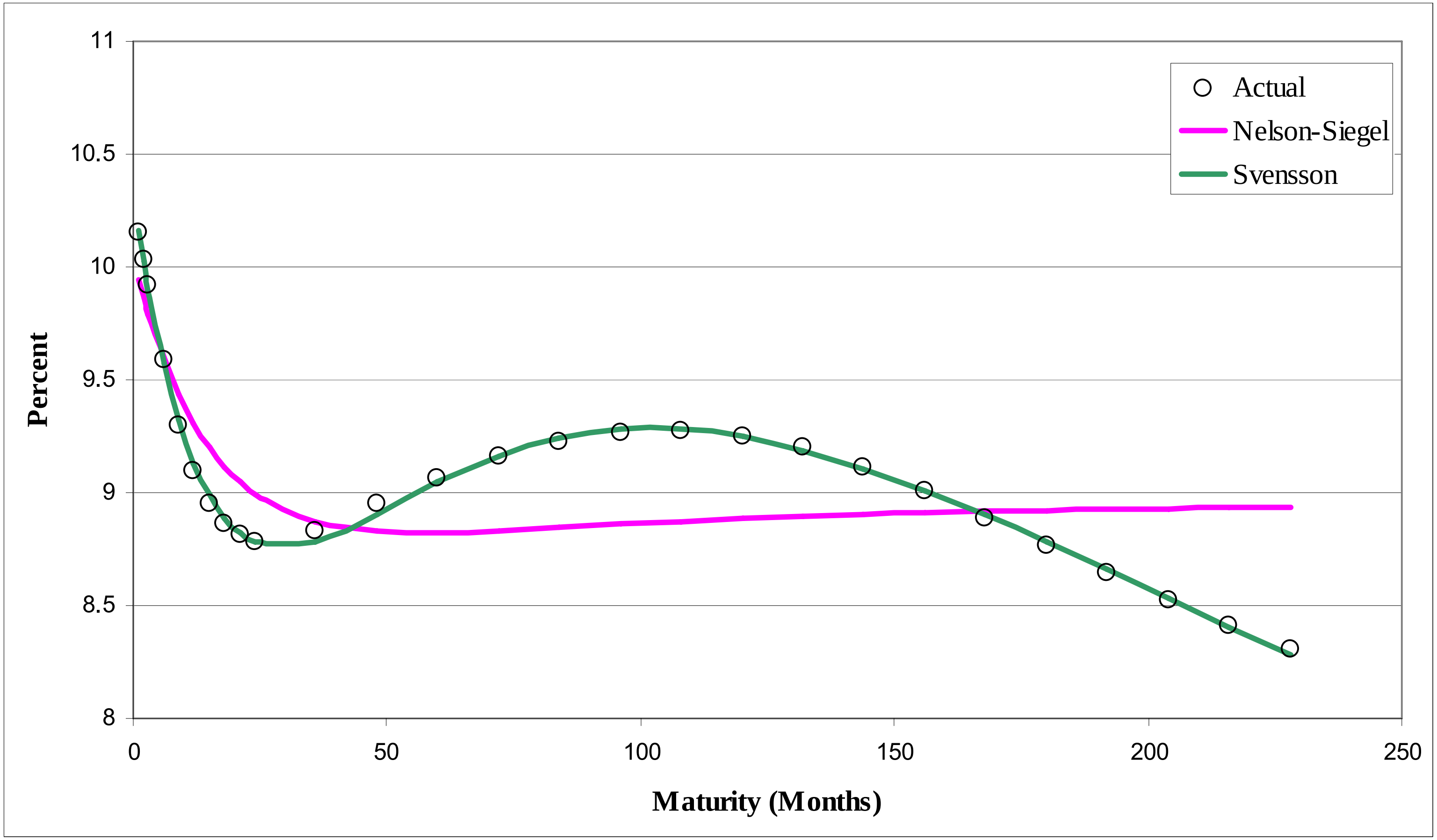
Yields-only model

- Four factor model: Svensson vs Nelson-Siegel



Yields-only model

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- Estimated *A* matrix (eigenvalues < 1)

	L_{t-1}	S_{t-1}	C^1_{t-1}	C^2_{t-1}	μ
L_t	**0.927** (0.050)	0.031 (0.039)	-0.049 (0.040)	0.006 (0.014)	**6.429** (1.054)
S_t	0.080 (0.049)	**1.024** (0.045)	**0.112** (0.039)	-0.016 (0.017)	2.357 (1.376)
C^1_t	**-0.251** (0.017)	**-0.193** (0.009)	**0.702** (0.053)	**0.071** (0.023)	-0.239 (0.391)
C^2_t	**0.231** (0.061)	-0.047 (0.118)	-0.134 (0.097)	**1.103** (0.030)	6.293 (4.028)

- Estimated *Q* matrix

	L_t	S_t	C^1_t	C^2_t
L_t	**0.473** (0.031)	0.007 (0.025)	-0.006 (0.117)	-0.048 (0.135)
S_t		**0.630** (0.032)	0.151 (0.133)	-0.045 (0.071)
C^1_t			**1.146** (0.152)	-0.022 (0.212)
C^2_t				**4.371** (0.678)

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Yields-only model

- Measurement errors

Maturity	Yields-only				Yields-macro			
	3 Factor		4 Factor		3 Factor		4 Factor	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	-0.0265	0.3633	-0.0147	0.4762	0.00499	0.45207	-0.00356	0.41848
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Yields-only model

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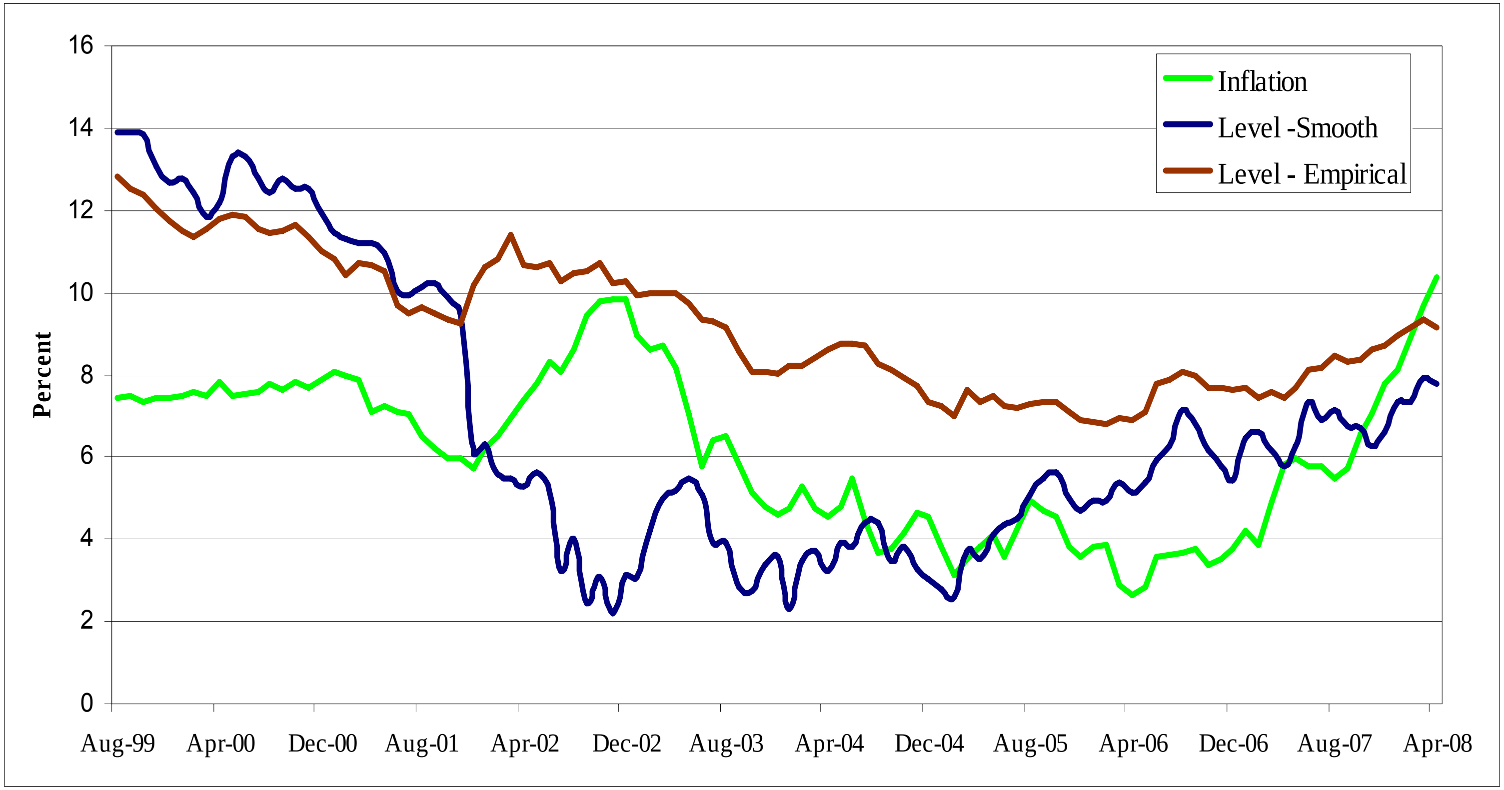
- Level



	Level -Smooth	*Level - Empirical*	*Inflation*
Level -Smooth	1		
Level - Empirical	0.682307979	1	
Inflation	0.388489617	0.778038131	1

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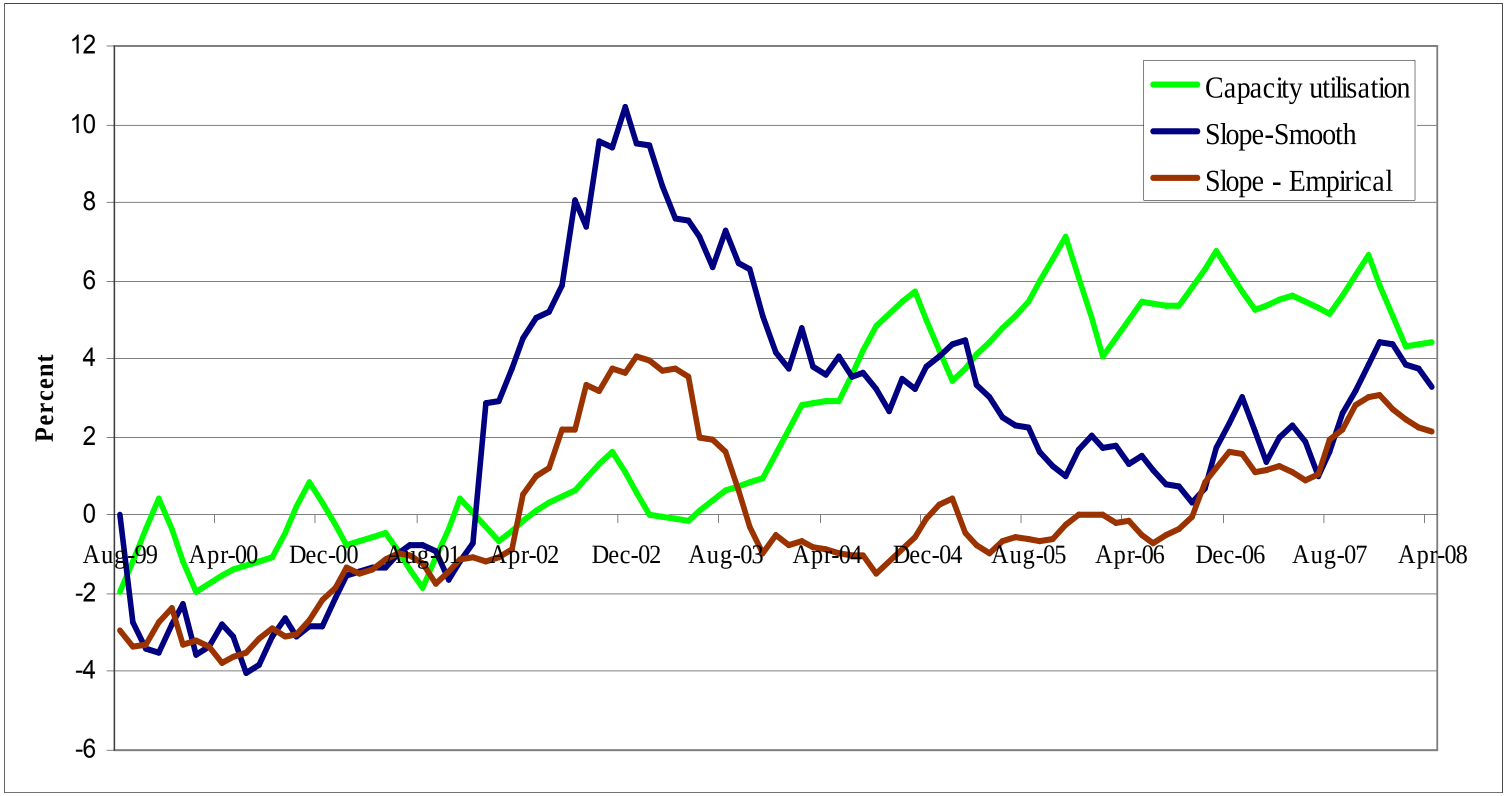
Yields-only model

- Slope



	Slope-Smooth	*Slope - Empirical*	*Capacity utilisation*
Slope-Smooth	1		
Slope - Empirical	0.84742317	1	
Capacity utilisation	0.224818122	0.36627818	1

Yields-macro model

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- Characterise the unobservable factors in terms manufacturing capacity utilisation, annual price inflation and repo-rate

$$(f_t - \mu) = A(f_t - \mu) + \eta_t ,$$

$$y_t = \Lambda f_t + \varepsilon_t .$$

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right]$$

$$f_t' = \left(F_1, F_2, F_{3,} F_4, CU_t, INFL_t, REPO_t\right)$$

  - parameter matrices are increased as appropriate

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Yields-macro model

ACTUARIAL SOCIETY OF SOUTH AFRICA

- Estimated *A* matrix

	L_{t-1}	S_{t-1}	C_{1t-1}	C_{2t-1}	CU_{t-1}	IF_{t-1}	RR_{t-1}	μ
L_t	**0.516** (0.036)	**-0.369** (0.034)	0.002 (0.033)	-0.020 (0.024)	0.044 (0.061)	0.109 (0.092)	**0.288** (0.097)	**6.427** (2.318)
S_t	**0.503** (0.114)	**1.303** (0.084)	**0.116** (0.040)	0.018 (0.028)	-0.046 (0.071)	-0.018 (0.109)	**-0.353** (0.060)	2.746 (2.535)
C_{1t}	**0.250** (0.079)	**0.217** (0.044)	**0.867** (0.066)	0.030 (0.048)	-0.045 (0.116)	-0.037 (0.161)	-0.303 (0.177)	0.785 (1.546)
C_{2t}	**1.622** (0.045)	**1.534** (0.102)	0.098 (0.082)	**0.907** (0.079)	**-0.533** (0.168)	-0.245 (0.303)	**-1.543** (0.298)	**7.747** (3.771)
CU_t	0.163 (0.120)	0.108 (0.091)	0.014 (0.023)	0.019 (0.016)	**0.938** (0.041)	-0.086 (0.062)	**-0.119** (0.048)	**83.127** (1.067)
IF_t	**0.416** (0.037)	**0.261** (0.045)	**0.057** (0.023)	**0.032** (0.014)	0.053 (0.038)	**0.953** (0.060)	**-0.206** (0.075)	**6.387** (1.427)
RR_t	**0.503** (0.062)	**0.324** (0.060)	**0.130** (0.019)	0.019 (0.012)	-0.029 (0.033)	**0.103** (0.050)	**0.534** (0.071)	**10.013** (1.091)

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Yields-macro model

- Estimated Q matrix

	L_t	S_t	C_{1t}	C_{2t}	CU_t	IF_t	RR_t
L_t	**0.525** 0.005	-0.001 0.036	0.000 0.087	0.000 0.112	0.000 0.077	0.000 0.019	0.000 0.042
S_t		**0.633** 0.096	0.000 0.060	0.000 0.171	0.000 0.036	0.000 0.051	0.000 0.011
C_{1t}			**1.512** 0.148	0.000 0.167	0.000 0.097	0.000 0.198	0.000 0.150
C_{2t}				**4.932** 1.249	0.000 0.100	0.000 0.195	0.000 0.165
CU_t					**0.210** 0.031	0.000 0.021	-0.001 0.011
IF_t						**0.199** 0.036	0.001 0.010
RR_t							**0.101** 0.013

Yields-macro model

- Measurement errors

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12	0.0298	0.3996	0.0188	0.5231	0.01876	0.33617	0.03652	0.33680
15	0.0248	0.4173	0.0212	0.5414	0.03973	0.34937	0.02497	0.35622
18	0.0205	0.4332	0.0240	0.5564	0.03634	0.36995	0.01856	0.38057
21	0.0171	0.4475	0.0264	0.5691	0.02913	0.38991	0.01649	0.40204
24	0.0138	0.4602	0.0267	0.5796	0.02419	0.40648	0.01736	0.41974
36	0.0066	0.4992	0.0115	0.6069	0.01982	0.42086	0.01931	0.43520
48	0.0144	0.5202	-0.0136	0.6194	0.01649	0.43349	0.02158	0.44886
60	0.0308	0.5285	-0.0352	0.6254	0.01308	0.44459	0.02343	0.46086
72	0.0472	0.5309	-0.0483	0.6288	0.00570	0.47664	0.02375	0.49547
84	0.0577	0.5323	-0.0521	0.6305	0.01297	0.49193	0.01631	0.51160
96	0.0585	0.5350	-0.0487	0.6310	0.02864	0.49666	0.00857	0.51633
108	0.0465	0.5398	-0.0417	0.6311	0.04441	0.49728	0.00546	0.51626
120	0.0202	0.5469	-0.0346	0.6317	0.05450	0.49787	0.00824	0.51527
132	-0.0202	0.5571	-0.0296	0.6330	0.05513	0.50052	0.01532	0.51502
144	-0.0733	0.5716	-0.0276	0.6349	0.04336	0.50604	0.02373	0.51603
156	-0.1358	0.5910	-0.0277	0.6367	0.01775	0.51479	0.03088	0.51807
168	-0.2038	0.6157	-0.0277	0.6383	-0.02155	0.52752	0.03526	0.52059
180	-0.2746	0.6454	-0.0268	0.6396	-0.07321	0.54507	0.03660	0.52321
192	-0.3467	0.6794	-0.0248	0.6408	-0.13401	0.56809	0.03615	0.52592
204	-0.4189	0.7169	-0.0217	0.6419	-0.20009	0.59659	0.03541	0.52861
216	-0.4900	0.7572	-0.0173	0.6431	-0.26898	0.62999	0.03514	0.53130
228	-0.5591	0.7997	-0.0112	0.6450	-0.33908	0.66765	0.03560	0.53413

Out-of-sample testing

- 1 year out-of-sample testing

Maturity	Four-Factor		Svensson - AR(1)		Four-Factor with repo-rate	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
3	-1.178	1.832	-1.530	2.301	-0.833	1.537
12	-0.862	1.335	-1.552	1.949	-0.512	1.066
36	-0.843	1.221	-1.866	1.436	-0.499	1.100
60	-0.985	1.324	-2.141	1.192	-0.647	1.272
120	-1.098	1.356	-2.348	0.967	-0.767	1.340
180	-1.207	1.176	-2.402	0.968	-0.883	1.114
228	-1.314	1.103	-2.439	1.067	-0.995	0.984

- 2 year out-of-sample testing

Maturity	Four-Factor		Svensson - AR(1)		Four-Factor with repo-rate	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
3	-0.437	1.869	-0.687	2.405	-0.193	1.486
12	-0.452	1.397	-0.855	2.022	-0.197	1.006
36	-0.865	1.101	-1.404	1.812	-0.611	0.708
60	-1.191	1.128	-1.781	1.849	-0.941	0.775
120	-1.401	1.170	-2.045	1.885	-1.157	0.867
180	-1.308	1.159	-1.966	1.863	-1.072	0.852
228	-1.197	1.141	-1.858	1.826	-0.967	0.824

Out-of-sample testing

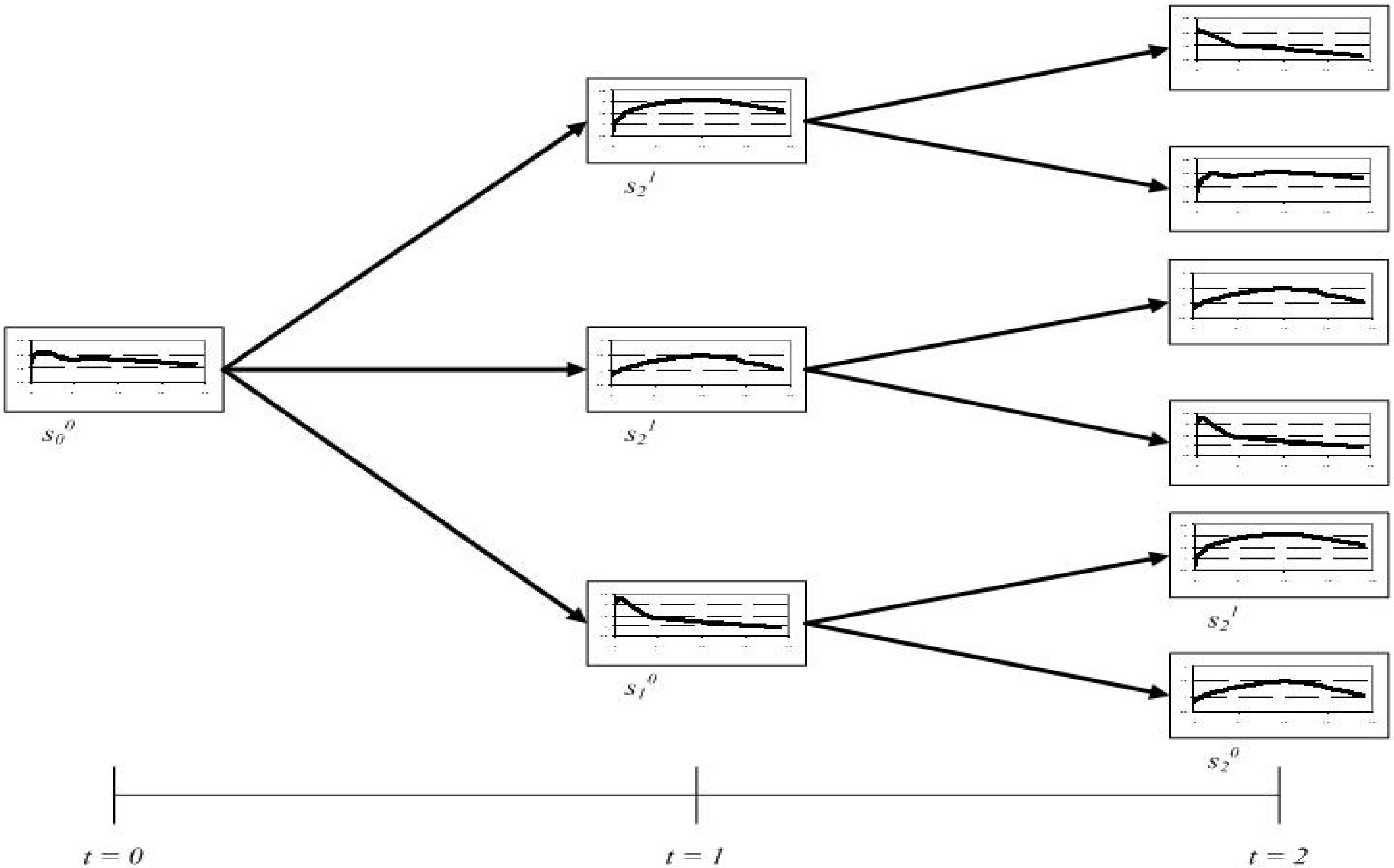
- 3 year out-of-sample testing

Maturity	Four-Factor		Svensson - AR(1)		Four-Factor with repo-rate	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
3	-0.991	2.311	-1.611	2.292	-0.631	1.959
12	-0.945	1.778	-1.845	1.835	-0.569	1.437
36	-1.453	1.302	-2.566	1.406	-1.081	0.965
60	-1.857	1.114	-3.048	1.249	-1.495	0.774
120	-2.141	1.011	-3.381	1.146	-1.791	0.679
180	-2.123	1.198	-3.285	1.241	-1.784	0.870
228	-2.082	1.384	-3.155	1.321	-1.750	1.060

- 4 year out-of-sample testing

Maturity	Four-Factor		Svensson - AR(1)		Four-Factor with repo-rate	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
3	-0.342	1.865	-0.925	1.809	-0.109	1.679
12	-0.401	1.210	-1.314	1.205	-0.149	1.023
36	-1.131	1.021	-2.286	1.058	-0.881	0.837
60	-1.642	1.034	-2.887	1.106	-1.400	0.854
120	-2.033	1.120	-3.333	1.167	-1.802	0.944
180	-2.070	1.421	-3.255	1.317	-1.846	1.242
228	-2.061	1.634	-3.117	1.392	-1.842	1.452

Scenario generation



Scenario generation

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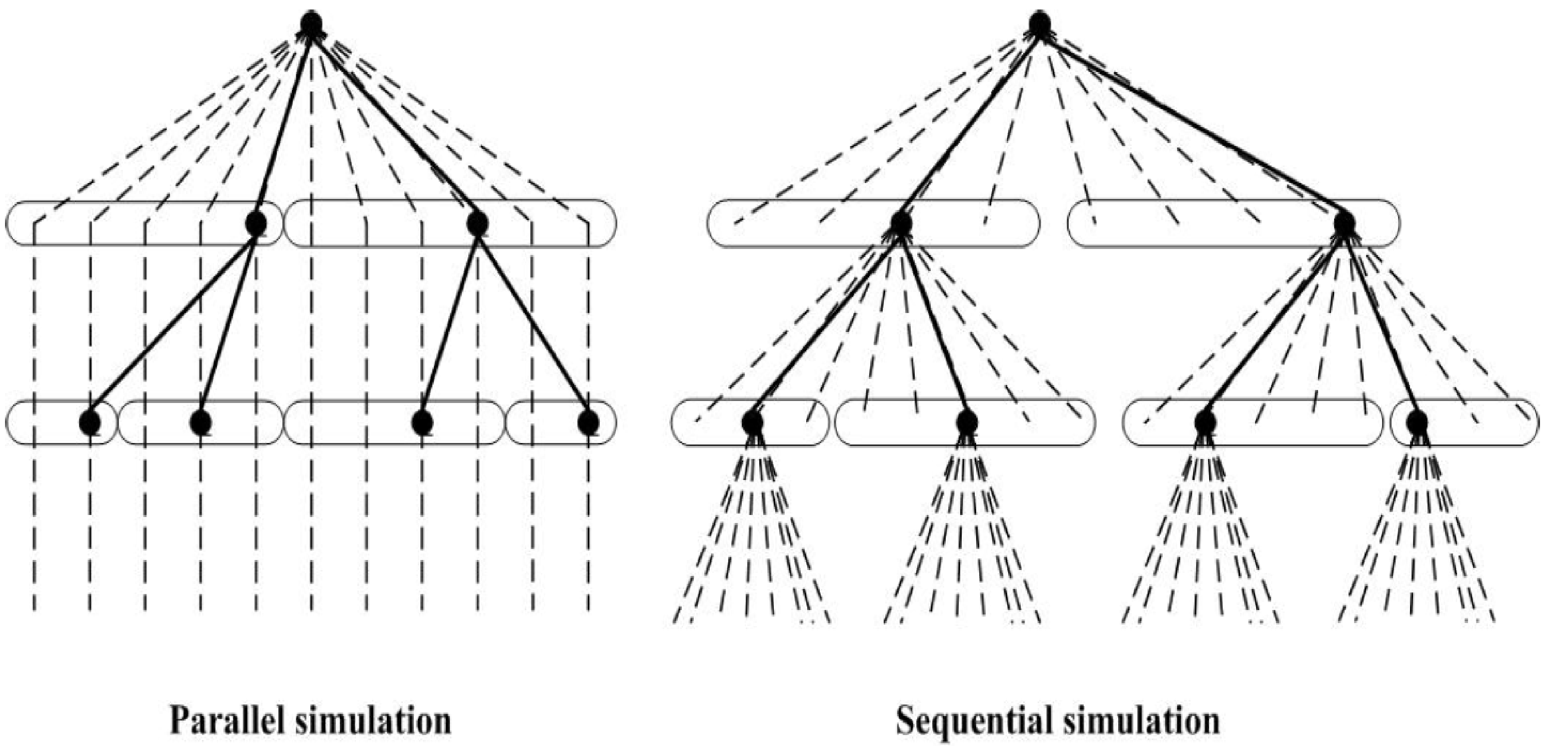
- Simulation and clustering approach (Gülpmar et al., 2004)



- Interest rate sampling algorithms (Chueh, 2002)

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Scenario generation

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Step 1: Create a root node group containing scenarios. Generate all the scenarios using Monte Carlo simulation and the four-factor yields-macro model. Each scenario is equally likely and consists of *T* sequential yield curves.
(In total *T*×*N* yield curves are generated.)

Step 2: For each group in the previous stage, calculate the mean scenario and calculate the *relative position* of each scenario with respect to the average scenario.

$$D = \sum_{\tau} \left(\frac{1}{\left(1 + y(\tau)\right)^{\tau}} - \frac{1}{\left(1 + y^{M}(\tau)\right)^{\tau}} \right)$$

Step 3: For each group, sort the scenarios in descending distance order and group them into equal sized groups.

Step 4: For each new group, find the scenario closest (in absolute value) to its centre, and designate it as the centroid. Assign a probability of to each centroid.

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Are these scenarios trees arbitrage free?

- Klaassen (2002) shows that arbitrage opportunities can be detected *ex post*.
- Filipović (1999) showed that the Nelson-Siegel family of yield curve models does not impose absence of arbitrage.
- Christensen et al. (2007) derives a class of arbitrage-free affine dynamic term structure models that approximate the Nelson-Siegel yield curve specification.
- Christensen et al. (2008) extends these models to include the Svensson yield curves.
- Their modification can be seen as a change of the slope of the yield curve.

Scenario generation

- We propose a method to reduce the presence of arbitrage *ex post*, without extending our models to the class of arbitrage-free models.
- This approach has no additional effect on the computational difficulty of the model estimation process or the data requirements.
- We also determine an overall slope change such that our yield curves passes Klaassen's test.
- After introducing a small bid/ask spread and transaction costs no arbitrage oppurtunities remain.

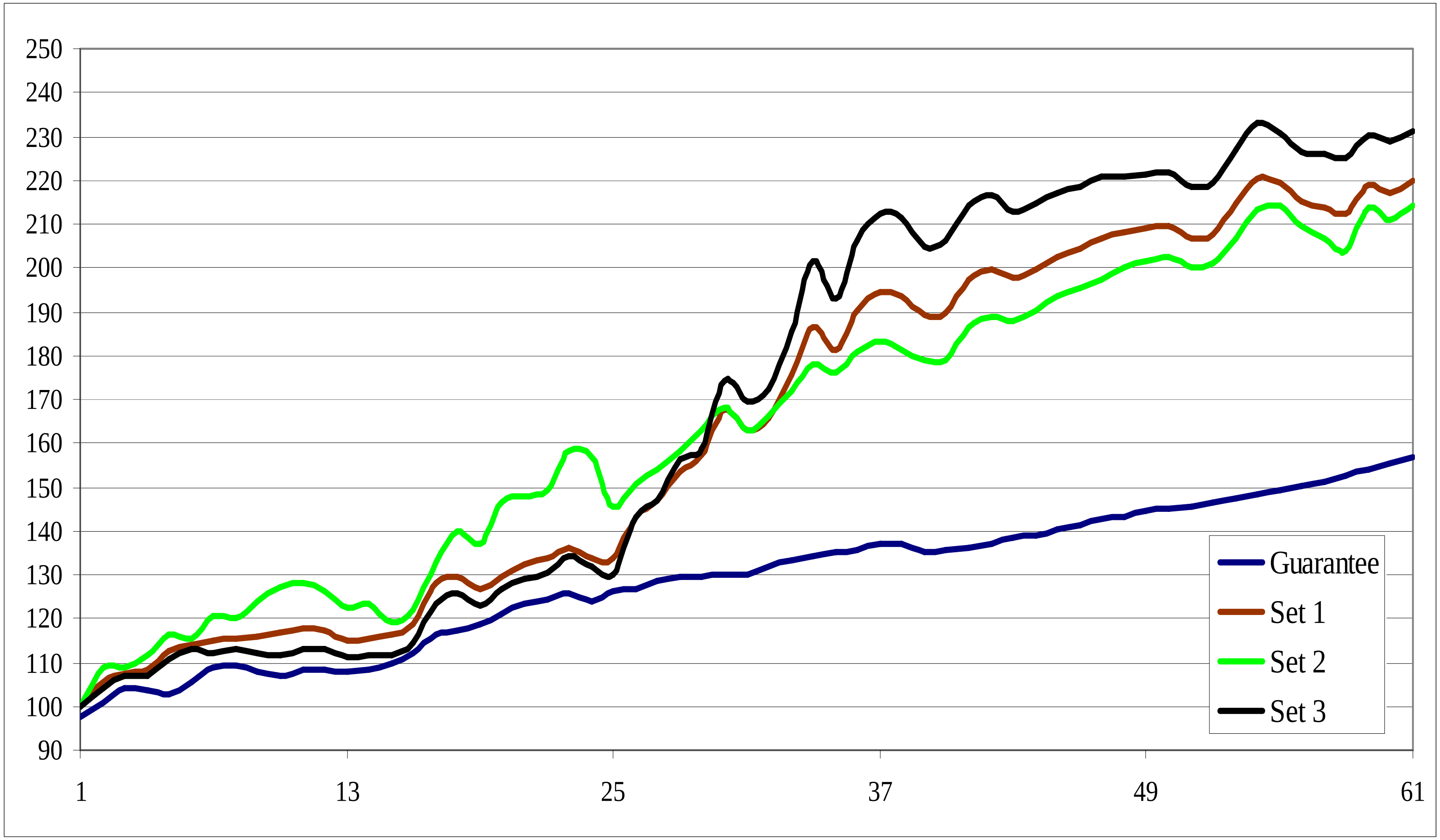
Back-testing

- Implemented the minimum guarantee return fund problem of Dempster et al. (2004)
  - An asset and liability management framework for a simple example of a closed-end guaranteed fund where no contributions are allowed after the initial cash outlay.
  - Demonstrate the design of investment products with a guaranteed minimum rate of return focusing on the liability side of the product.
- Different tree structures

Year	Set 1	Set 2	Set 3
April 03	5.5.5.5.5 = 3125	13.4.4.4.4=3328	200.2.2.2.2 = 3200
April 04	8.8.8.8 = 4096	15.6.6.6 = 3240	400.2.2.2 = 3200
April 05	15.15.15 = 3375	30.10.10 = 3000	400.3.3 = 3600
April 06	56.56 = 3136	160.20 = 3200	800.4 = 3200
April 07	3125	3328	3200

Back-testing

- Back-testing results



Conclusion

- Estimated and characterised the South African term structure with respect to macro-economic variables
- Estimated a model that incorporates four yield curve factors (level, slope and two curvature factors) and macro-economic variables (real activity, inflation and the stance of monetary policy)
- Model fits the term structure reasonably well in-sample and performs reasonably well in out-of-sample forecasting
- Better performance can be realised by including the investors expected view on the repo-rate
- Proposed a parallel simulation approach for yield curve scenario tree generation
- Performance is measured by out-of-sample back-testing in terms of the value of a fixed income portfolio optimization problem described in the literature
- The results demonstrate a reasonably sound way to generate stable yield curve scenario trees

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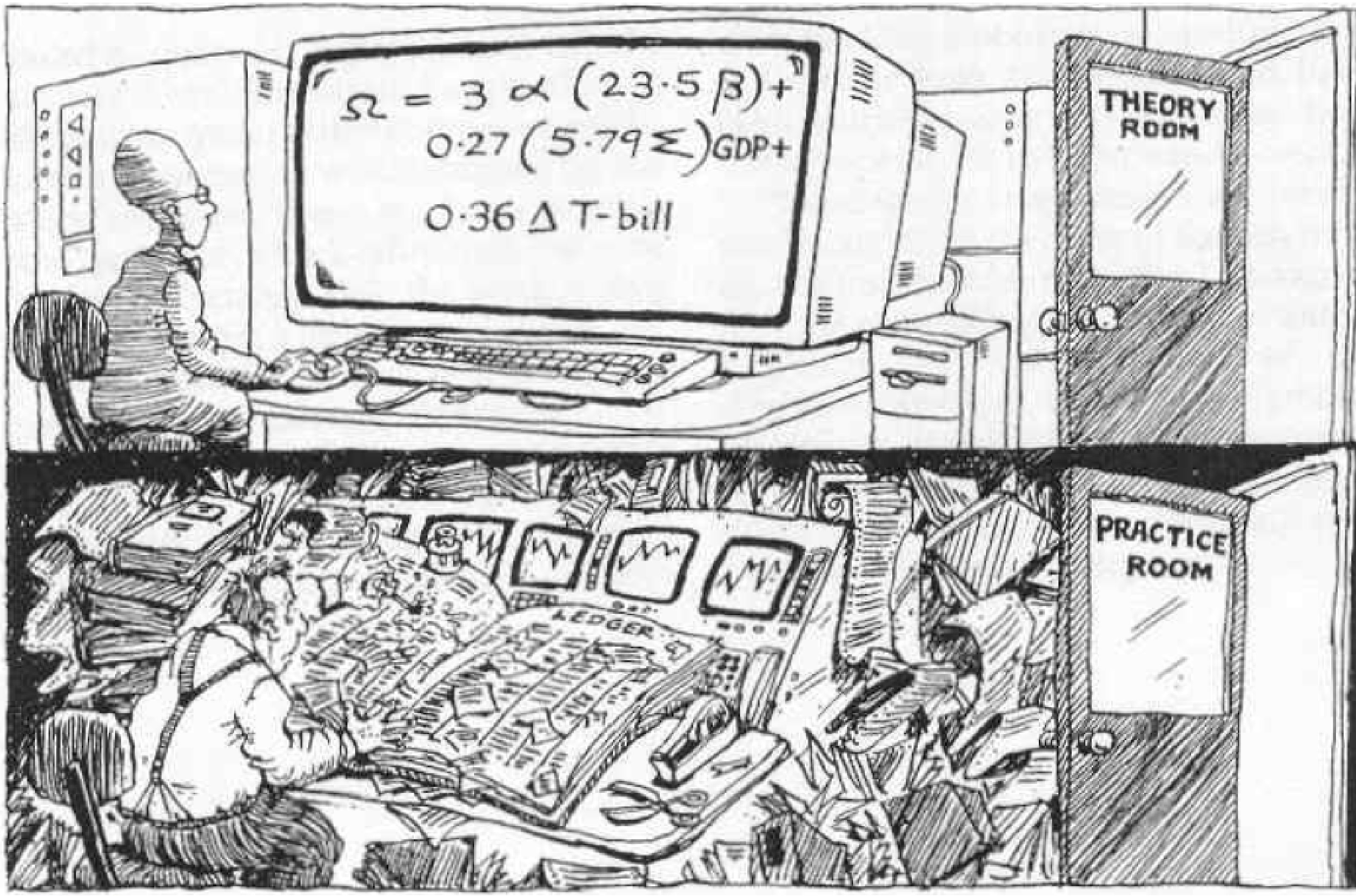

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Thank you

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“**Shortcuts can slow you down**”, Jack Johnson