Optimal Annuity Strategies After Retirement

The secret to living well is to die without a cent in your pocket. But I miscalculated, and the money ran out too early.

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Abstract

There is a shortage of South African research on how to make optimal use of one's retirement payout. Most individuals purchase either a guaranteed life annuity or an investment linked living annuity, without necessarily quantifying the potential benefits and potential risks of all the options available to them.

Modern stochastic techniques now enable us to project future income and, where applicable, capital levels for the various annuity types. Our paper does this for guaranteed life, living and with-profits annuities, comparing them on a purely financial basis. In doing so we also try to establish optimal combinations of asset allocation and drawdown rate for the living annuity.

We then go on to look at whether it makes financial sense for pensioners to hold a living annuity during the early years of their retirement, and move to a guaranteed life annuity in their later years.

Executive Summary

This paper analyses which of the main annuity types in South Africa are expected to provide the most future lifetime income to a 60 year old male retiring on 31 December 2007. The results naturally rely on the assumptions made and methodology used, and by no means aim to provide generic advice to all retirees.

The main conclusion drawn about living annuities is that if the retiree wants to try and obtain a level of income that is comparable to a guaranteed life or with-profits annuity, he is almost certainly going to be faced with a decreasing income stream over time. This is unlikely to meet his retirement needs.

On the other hand, if the retiree wants to try and maintain or grow his living annuity income, he needs to select a low initial drawdown rate. We find that to have a high probability of maintaining the Rand amount of his income, the retiree should not have an initial drawdown rate of more than 7.5% of the living annuity capital. This should reduce to 2.5% or 5% (depending on risk-preference) if the retiree wants a growing income.

We show that customising or varying the drawdown percentage over time can help in increasing the chance of obtaining the desired income pattern.

In terms of asset allocation, it seems that a more conservative investment portfolio (25% in equities) is optimal. An interesting finding is the path-dependence of income aspect of the more risky portfolios – disinvesting income during periods of negative returns reduces the ability to benefit from any subsequent positive returns.

The guaranteed (both level and 5% increasing) and with-profits annuities seem to be far superior to the living annuity from a risk-adjusted return point of view - they provide similar / greater expected future lifetime income at far lower risk. Living annuities of comparable risk provide far lower expected lifetime income.

Switching from a living to a guaranteed annuity during retirement can add value (both in terms of increasing income and reducing risk). However, because of the relatively low net (after charges) living annuity returns and the downward sloping initial yield curve, it is still likely to be optimal to purchase a guaranteed annuity at retirement (in other words, not delay annuitising).

What is not clear is whether one should choose the with-profits or one of the guaranteed annuities. This will eventually come down to individual risk-preference – the with-profits annuity is expected to provide around 14% greater lifetime income (in present value terms) than a guaranteed annuity, but has a 20% chance of providing 17% less income (again in present value terms). Alternatively, it has a little more than 50% chance of not being able to grow the income by 5% p.a.

Inflation is also an important consideration in South Africa. Although not quantified, the with-profits annuity should be able to offer a great degree of inflation protection in times

of high inflation than a guaranteed annuity (apart from an inflation-linked one, which currently offers very low initial income levels). In these situations a level guaranteed annuity would be a very poor option.

In conclusion, we return to the financial advice side. What is certainly true is that an optimal annuity strategy cannot be determined without looking at the personal circumstances of the retiring individual. How wealthy are they? How financially sophisticated? How long are they likely to live? Do they have dependants that need to be provided for? How much control over their retirement capital / income do they want?
Possibly most importantly, what are their income requirements? An annuity strategy alone cannot necessarily solve all of a retiree's needs, particularly if he / she has not retired with enough capital.

Hopefully, however, this research will help inform advice so that those retiring can make the most out of their hard earned capital.

Acknowledgements

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1 Background / Intention of Research

Despite the title, this paper does not aim to be an authoritative guide on what to do with one's retirement savings. That is a complex field of financial planning that requires in-depth knowledge of the retiring individual's circumstances, needs and preferences.

This paper does, however, intend to highlight the need for research in South Africa into how various options available to a retiree are expected to compare on financial terms.

For example, some of choices faced by an individual retiring include:
- Purchasing a guaranteed life annuity from an insurer, which provides a level income for the rest of the individual's life
- Purchasing a guaranteed life annuity, but with a built-in annual income increase
- Investing in a living annuity, choosing which portfolios to select and how much income to withdraw
- Purchasing a with-profits annuity

Taking our local regulations, economic conditions and product types into account, is it possible to get clear guidance on:
- Where is he likely to get the most long-term income?
- Does the current level of interest rates matter?
- How much equity exposure in a living annuity is optimal?
- What living annuity drawdown rate is optimal?
- Should he be looking to vary this drawdown rate over time?
- Should he be looking to start off in a living annuity and move to a guaranteed life annuity later on?

These are without doubt key questions in determining a (post) retirement plan.

This paper's main focus is thus on **which annuity strategy is expected to provide the most income to a retiree over the rest of his life, and what the risks around this are**. In doing so we use the case of a 60 year-old male retiring with R500,000 on 31 December 2007.

Although not strictly involved in this field, the authors had both a personal interest in the topic and (possibly more importantly) access to modern stochastic asset projection models. This allowed us to make a first stab at answering the questions shown above.
The research can certainly be refined and improved on

Hopefully this work will be able to form a base that can be developed in future, ideally influencing financial advice and benefiting those who retire.

2 Comparison of Annuities Types

2.1 Annuities Purchased in the SA Market

The term “annuity” is used throughout this paper to refer to **compulsory purchase post-retirement income annuities** (as opposed to [pre] retirement annuities, voluntary purchase income annuities or US style “annuity” investment products). The types most commonly purchased in South Africa (and those examined in this paper) are:

2.1.1 Guaranteed Life Annuity

Often also called an immediate life annuity or single premium immediate annuity (with the act of purchasing one often just simply called “annuitisation”), this is probably the easiest annuity for the retiring individual[1] to understand. In it’s simplest form it pays a guaranteed income to the annuitant until death.

The life insurance company that offers or underwrites the annuity carries the risk that:
1. Investment returns are not sufficient to provide the income for the annuitant’s lifetime (investment risk)
2. The annuitant lives longer than expected (mortality risk)

It does so by using the traditional insurance concept of pooling lives, as well as typically following a matching (i.e. low risk) investment strategy.

To better meet their needs, the annuitants generally have the option to:
1. Ensure their beneficiaries receive something on their death
    - A guarantee term can be added – the annuity income will be paid for a certain minimum term (e.g. 10 years) irrespective of whether the annuitant is alive or not
    - A joint-life option can be taken out – on death of either the annuitant or their partner, the annuity income will continue to be paid to the remaining individual
    - Life cover can be purchased – a lump sum amount is paid on death of the annuitant. When packaged with the annuity, this cover is generally available without medical underwriting (the life insurer loses on the life cover side if the retiree is in poor health, but gains on the annuity side).
2. Ensure their income increases over time
    - Typically the annuitant can specify a fixed percentage annual increase (which is guaranteed) or may even be able to have their income increase with an official inflation index. In both cases the initial level of guaranteed income is reduced to fund the cost of the increases.

[1] The terms “retiree”, “annuitant” and “investor” are used interchangeably in this paper

2.1.2 Investment Linked Living Annuity

The living annuity (potentially a misnomer) provides the retiree with a greater degree of freedom than that afforded by the guaranteed life annuity. Each year the retiree is able to select the amount of income he wishes to receive, and this is provided via regular withdrawals from his living annuity capital. Under the current South African legislation, this withdrawal rate is limited to between 2.5% and 17.5% p.a. of the start of each year's capital value.

The intention behind the drawdown limits is to try and ensure annuitants use the product for which it is intended (and for which pre and post retirement tax benefits have been granted) – to provide a lifelong income after retirement. There has been a lot of discussion in the industry in recent times around the suitability of the limits (in low interest rate environments it is difficult to justify that a 17.5% drawdown rate will provide a lifelong income), and they have recently been reduced from 5%-20%.

In some overseas markets, the drawdown rate is limited to the income available from a guaranteed annuity. This is not the case in South Africa, although new disclosure codes require the retiree to be warned about choosing a drawdown rate above the income level offered by a 5% increasing guaranteed annuity.

In a living annuity the retiree is also able to choose the assets in which his capital is invested. Most modern living annuity products offer a wide variety of investment portfolios, typically unit trusts funds from various providers. Some providers offer specialist portfolios designed for income provision. The asset allocation should comply with prudential investment guidelines (where at most 75% is invested in equities).

On the death of the retiree, the remaining capital in the living annuity portfolio is not forfeited, as is the case with a guaranteed annuity. Instead, it transfers to the retiree's nominated beneficiaries. (Hence the often misunderstood name "living annuity" – the capital "lives on" after the death of the annuitant.)

Depending on the portfolio and product choice, there may be varying levels of asset management, intermediary advice and expense charges.

The retiree bears the full investment and mortality risk, and possibly also some expense risk (risk of living annuity charges increasing).

The popularity of living annuities has risen incredibly over the last decade in South Africa.

In the US, a living annuity strategy is often called self-annuitisation and in the UK an income drawdown.

2.1.3 With-Profits Annuity

The with-profits annuity is typically sold in the institutional market (i.e. to pension funds rather than individuals), but some life companies do sell a retail version of it. It provides a guaranteed income for life with some investment participation in the form of increases to the annuity income via annual bonus declarations. These bonuses are derived from the annuity consideration being invested in an underlying portfolio, typically with a balanced asset allocation and smoothing of returns.

In terms of the risk-return trade-off, the life company takes on most but not all of the investment and mortality risk (i.e. this falls between a guaranteed life annuity and a living annuity).

2.1.4 Other Types and Overseas Markets

Different annuities sold in other markets were not examined in detail, as they were not central to the objectives of this paper. However, it is important to note where other markets have the same types and there is a clear preference for one of them. Section 2.3 on utility discusses some of the reasons retirees overseas have historically chosen not to purchase guaranteed life annuities but instead "self-annuitise" (enter into a living annuity arrangement).

2.2 Comparison on Financial Terms – Null Hypotheses

Starting out, we felt the following factors would be the most important in determining which annuity strategy would provide the most income over an annuitant's lifetime:

2.2.1 Mortality / Capital Effects

By not transferring the mortality risk to the life company, the living annuity investor has to take care in managing the longevity of his income.

If he knew how long he was going to live (i.e. there was no mortality risk), he could draw his capital down to zero over his lifetime, maximising his income.[2] He doesn't, however, have this perfect foresight, and even if he did, the drawdown regulations in South Africa prevent him from running his capital down completely.

The 17.5% maximum drawdown also means that if living annuity capital starts to decline and the drawdown rate is already at the maximum, the retiree's income will decrease. This is the opposite of what retirees generally want from their retirement income, which is for it to grow over time with inflation.

It is thus suggested that the living annuity investor cannot risk his capital depleting. This in turn means that, all other things being equal, **a living annuity should provide a lower total lifetime income than a guaranteed life or with-profits annuity** (whose income / annuity rate implicitly assumes the capital is fully depleted over the retiree's expected lifetime). In some of the first research on this topic, Yaari (1965) puts forward that risk-averse retirees without a bequest motive should annuitise fully.

This is especially so for older individuals, where their low life expectancy means the "capital component" of the guaranteed life / with-profits annuity's income is large. For example, according to the SAIL98 Mortality Table used in this paper there is 14.3% chance that a 90 year-old female will die in the next year. If 1,000 such females enter into a one-year life annuity agreement by investing R100 each in a cash pool yielding 6.5%, the funds will grow to R106,500 by the end of the year. Of the starting 1,000, 143 are expected to die leaving 857 survivors. This leaves R106,500/857 = R124.27 per survivor. This is a return (expected) of 24.3%. This far exceeds the risk free return of 6.5% (or perhaps any risky return) because the annuitants have seceded control of assets in the event of death.

The 24. 3% investment return contains 6.5% of the bank's money and a healthy 17.8% of **"mortality credits"**. These credits represent the capital and interest "lost" by the deceased and "gained" by the survivors. Here the average non-survivor has forfeited any claims to the fund.

What if the R100,000 was invested in the stock market and the stock market headed south 20%? Each survivor this time only earns R80,000/857 = R93.35. This is a return

[2] It must be noted that it isn't necessarily true that the retiree wants to minimise the capital remaining on his death. One reason for buying a living annuity is that on death any remaining capital goes to the retiree's beneficiaries. This is expanded on in the utility aspects section.

(expected) of –6.7%. Such is the power of mortality credits. They subsidise losses on the downside and enhance gains on the upside.

Of course, in practice, annuity contracts are for life, and these credits are spread and amortised over many years of retirement. But the basic insurance economics underlying the contract are exactly as described.

This is something that has been picked up on by some international researchers (for example, as in Milevsky (1998) and Milevsky and Young (2002), who advocate that individuals should switch from living to guaranteed life annuities when they reach an age at which the mortality credits / capital effect begins to dominate.

2.2.2 Cost / Charge Effects

The costs / charges incurred by each annuity type are detailed in section 3, but intuitively:

- The guaranteed life annuity (in a competitive, commoditised market) should have the lowest charges
- The with-profits annuity (with some risky asset class exposure and capital charges for guarantees) should be next best
- The living annuity (charging retail asset management and admin fees as well as for ongoing advice) should have the highest deductions.

2.2.3 Asset Allocation / Return Effects

Intuitively, a higher equity allocation should enable the living and with-profits annuity to earn better long-term returns, providing a higher overall income than a guaranteed life annuity.

Conversely, a living annuity with a very conservative asset allocation (predominantly fixed interest investments, similar to those typically backing a guaranteed life annuity) should be inferior due to its higher charge deductions and inability to use up its capital efficiently (lack of mortality credits).

In many scenarios it isn't necessarily obvious which of the three effects (capital vs. cost vs. asset allocation) will dominate.

2.2.4 Current and Expected Future Interest Rates

An important aspect is whether the current levels of interest rates are likely to persist in future. Over recent periods of low interest rates there have been theories that one should not lock into low guaranteed life annuity rates. Certainly this would have been detrimental in South Africa the last few years. Now that interest rates are significantly higher some market commentators are beginning to say the opposite.

In theory, however, if one purchases bonds upfront in a living annuity and the yield curve unwinds as expected (for instance, rates reduce in future), one should come out with a similar gross investment return as in a guaranteed life annuity. The key difference in practice is that living annuities are generally not invested in portfolios that hold bonds purchased upfront to term. The bond holding changes over time as other investors come in / out and when one has to disinvest capital to provide the required income. (Unlike the insurer does for a life annuity, the living annuity investor is unlikely to conduct a cashflow matching exercise).

There can also be an impact on expected risky asset class returns (e.g. equities). If one assumes these earn a risk-premium above risk-free, and the risk-free rate is expected to fall then so will risky asset returns. In this situation, locking into guaranteed life annuity at retirement may be advantageous.

This is also an important point when one considers switching to a guaranteed life annuity from a living annuity at some future point. A downward sloping yield curve implies lower guaranteed life annuity rates in future, which can reduce the expected benefit of switching.

A possibly more appropriate reason for not locking into a guaranteed life annuity when interest rates are low is that the level of income may not meet the annuitant's living expenses. One might decide to park the funds in a living annuity (or delay retirement) until life annuity rates improve. In doing so one takes the risk of:

- Rates not improving in future,
- Rates improving, but only as a result of inflation increases (and therefore real income not improving), and
- Depending on the investment strategy and drawdown rate, being faced with a depleted capital position.

2.2.5 Risk Effects

Any expected return gains from the living and with-profits annuities needs to be weighed up against the additional risk borne by the annuitant. The risk aspect is important, and is a key component of this paper.

One tends to think that this depends more on the annuitant's attitude to risk (and is therefore qualitative, or utility based) but examining extreme situations we can see this is not entirely true:

- With current return expectations, if one draws down 17.5% p.a. of one's living annuity the risk of running out of money has to be too great [3]
- Similarly, gut feel tells us that investing 100% of one's living annuity in equities has to be too risky

Both of these situations can be described by **the probability of having insufficient income being too high**. Given a definition of "insufficient income", projecting future returns stochastically allows a value to be placed on these probabilities.

[3] Although as previously explained, the maximum drawdown rules mean one can never totally run out of money

The prior examples also indicate that not only is the asset allocation and investment return scenario important, but also the living annuity drawdown rate.

Note that only investment risk and not mortality risk is modelled stochastically in this paper. Mortality rates are assumed to follow expectation. One could model mortality rates stochastically, and build the likelihood of living longer than expected into the probability of having insufficient income.

2.2.6 Switching Annuity Types During Retirement

As explained, there is a case for initially investing in a living annuity and then switching to a guaranteed life annuity (or with-profits annuity) when one is older (when the capital effect of the guaranteed annuity is high).

Some researchers (Blake et al, 2003; Milevsky, 1998; Milevsky and Young, 2002) have suggested that such a mixed strategy would be attractive. It potentially enhances income early on (through earning an equity risk-premium on one’s living annuity investment), and also adds the capital usage / insurance feature later in life.

Milevsky and Young (2002) discuss what they call the “real option to delay annuitisation” embedded in the decision to annuitise. Logically, deciding to purchase a guaranteed annuity is like exercising an American option where exercise is dependent on mortality effects. Exercising such an option should be at a time when its time value is worthless. Thus, if we account for future mortality and uncertain investment returns, the embedded option provides an incentive to delay annuitisation until the option value is nil. The option is said to be real because it is not tradable and is personal i.e. its value depends on the retiree’s subjective mortality rate and degree of risk aversion. Milevsky and Young (2002) show that this option is still valuable as long as the discounted expected utility from delaying annuitisation is more than that of annuitising immediately. One should switch when the discounted expected utility from purchasing a guaranteed life annuity is the same as that from deferring annuitisation.

Looking at this in financial terms, deferral makes sense (the option to delay annuitisation has value) as long as the stochastic return from invested wealth exceeds the mortality-adjusted risk free rate i.e. as long as the living annuity’s return exceeds the guaranteed life annuity’s mortality credits.

Using the mortality and life annuity calculation bases detailed in sections 3.3 and 3.4 we can estimate the mortality credits for different starting ages. Here we have defined these as the return above risk-free implied by the change in guaranteed life annuity rates over the next 10 year period. The table overleaf provides an estimate of these mortality credits at different ages (with the calculation method explained in Appendix A).

Estimate of Mortality Credits at 31 December 2007

	Males	**Females**
Age	**Mortality credits (bps)**	**Mortality credits (bps)**
55	10	-34
60	57	-10
65	101	26
70	148	76
75	198	142
80	236	207
85	239	238

At age 60, looking at males, the mortality credits are about 57 basis points i.e. the guaranteed life annuity provides a 0.57% increase in return (yield pickup) above risk-free from ages 60 to 70 due to capital usage.

To put this number in context, if a recent retiree or their advisor thinks they can earn 57 basis points more than the risk-free rate after charges over the next 10 years they are better off not purchasing an annuity today and managing the money via a living annuity. This "benchmark" or "hurdle" rate of return can be used to assess whether one should annuitise now or later.

One can see how the mortality credits increase over time, and that they are lower for females than males (so much so that females in a living annuity can even earn less than the risk-free rate at young ages). This makes intuitive sense.

The key consideration is thus what rate of return can be earned in the living annuity, and at what stage the life annuity mortality credits outweigh this.
For example, if one assumes that the investment portfolio can earn a gross return of 3% p.a. above risk-free and total living annuity charges are 2% p.a., this means a net return of 1% p.a. above the risk-free rate. **On this basis males should annuitise at around age 65 and females between 70 and 75.**

There are clearly risks to this strategy:
- Living annuity returns might be poor, depleting capital
- Future annuity rates might turn out to be lower than expected
- The client might not switch at the optimal point (because of lack of knowledge or advice)
- It may not be efficient from a cost point of view

2.3 Utility Aspects

Much of economics literature has documented extremely low levels of annuitisation (i.e. not purchasing a guaranteed life annuity) among retirees. This result is inconsistent with the results of a standard Modigliani life-cycle model of savings and consumption. Yaari (1965) used this model to demonstrate that in a life cycle with no bequest motives, all consumers will hold life annuities as opposed to liquid assets or marketable wealth. The rationale behind Yaari's conclusion is that life annuities dominate all other assets because of the capital usage / mortality credits previously discussed.

Despite this, there is little evidence of retirees choosing to purchase life annuities. The reader can be referred to the work of Modigliani (1986), Friedman and Warshawsky (1990), Mirer (1994), Porteba and Wise (1996) and Brown (1999, 2001) among others, who have pointed out that very few people choose to annuitise their wealth once they reach retirement.

A number of reasons or theories have been advanced to explain this. These include

1. The desire to leave capital to dependants.
   Typical life annuity structures mean the annuitant forfeits their capital at purchase; a negative feature for retirees who wish to leave a bequest.
   Empirical evidence of bequests in overseas markets appears to be light. In the UK, Gardner and Wadsworth (2004) report that of those retirees who choose not to annuitise, less than 40% are motivated by bequest motive or concern about early death.

   Rusconi (2006b), however, suggests that in South Africa bequest motives may be the key driver of annuity choice. At the time, an informal survey by a leading adviser of retirement funds found that 75% of retiring South Africans choose a living annuity rather than a life annuity. National rates may be lower as living annuities tend to be the preserve of the more financially savvy retirees.

2. Flexibility.
   Gardner and Wadsworth (2004) found that the dominant reason not to annuitise was desire for flexibility. Nearly three-quarters of the respondents to a survey of those a few years into retirement cited flexibility as one of the reasons for not annuitising.
   Living annuities give the retiree choice over both investment allocation and income level, subject to certain constraints.

3. Level of interest rates
   Rusconi also suggests that falling annuity rates may have lent credence to the perceived poor value of annuities.
   The fact that SA living annuity drawdown limits do not reduce when interest rates fall might drive retirees requiring a certain monthly income to a living annuity.

4. Remuneration incentives
   Living annuities give the financial adviser the ability to manage the retiree's investment portfolio and earn trail commission / ongoing advice fees.

5. High actuarial loadings arising from adverse selection (Mitchell et al., 1999). Purchasers of annuities tend to live longer than the population average. Annuities therefore provide better value for money to individuals who are optimistic of their life expectancies. Providers of life annuities recognise this self-selection bias and consequently reduce annuity rates.
   Evidence in South Africa, however, does not seem to support this. Rusconi (2006a) suggests that "value for money is reasonable, competition strong and charges acceptable". This is also tested in this paper.

6. Alternative support from government via social security payments and or grants. (Munnell et al. 2002)
   This is currently not very applicable in South Africa.

7. Better long-term returns and greater overall income
   The living annuity can allow retirees to earn a risk-premium above the matched returns of the guaranteed life annuity.

8. Alternative support from family members (Brown and Poterba, 2000; Kotlikoff and Spivak, 1981). Families provide an opportunity to pool mortality risk in that if one family member dies, joint wealth passes to the survivors.
   This reason may also very well not apply in South Africa.

Having said this, there are definitely non-financial reasons why a retiree may prefer a guaranteed life annuity. These include

1. Security
   The retiree can secure a guaranteed lifetime income, and does not have to take the risk of poor investment returns or living longer than expected.

2. Simplicity
   The retiree receives his guaranteed income every month. He does not have to concern himself with managing an investment portfolio.

3. Greater certainty over bequests
   The retiree is unlikely to know in advance how much living annuity capital will remain on his death. The guaranteed life annuity's death payment options (joint-life, life cover, guarantee terms) can provide more certainty over this.

This last point is an important one. It isn't adequate to choose a living annuity purely because one wants to leave a bequest to dependants. Other annuity types may also have these options (and may even suit the retiree's needs more), and one should evaluate them all before choosing an annuity type.

Because this paper is about quantifying the benefits from each annuity type, we now look at whether there are ways to measure these utility aspects.

2.3.1 The Classical Utility Model

In financial economics literature, when analysing problems dealing with decision making under uncertainty, it is common to hypothesize the existence of a von-Neuman Morgenstern utility. According to von-Neuman Morgenstern, each individual has a measurable preference among various choices available in risk situations. This preference is called his utility. By suitable questioning we can determine for each individual a relationship between utility and wealth, which is called his utility function. In any decision involving risk, for example optimal asset allocation, insurance and consumption patterns, the rational man will choose that alternative which maximises his utility subject to exogenous resource and budget constraints.

In analysing the different annuity strategies, the classical approach would have been to consider which strategy or combination of strategies maximises the following dynamic stochastic optimisation problem, Milevsky (1998):

$$\max_{\{C_t,\alpha_t,a_t\}} E\left[\int_0^T e^{-\rho t}U(C_t)dt + e^{-\gamma T}B(W_T)\right]$$

where $U(C_t)$ denotes the instantaneous utility of consumption

ρ is the personal discount rate

$B(W_T)$ is the utility from bequest

γ is the bequest discount rate

T is the stochastic time of death

W_t is the level of marketable wealth e.g. living annuity fund size at time t

α_t is the asset allocation vector

a_t is the amount of annuities

Implicit in $U(C_t)$ and $B(W_T)$ is a functional form that will involve at least two parameters, namely the marginal utility of consumption and bequest.

Blake, Cairns and Dowd (2000) consider a negative exponential utility function to demonstrate that the best annuity strategy is one that pays a mortality bonus to the policyholder in return for which the residual fund reverts to the life assurer or pension provider on the policyholder's death. They model the pensioner's discounted utility as

$$V = \sum_{t=0}^{K} e^{-\beta t} J\big(P(t), D(t)\big)$$

where K is the curtate future lifetime so that for a 65 year-old at retirement $65 + K$ is the rounded down age of the retiree at death

β is a discount rate reflecting the pensioner's time preference.
$J(....)$ is a utility function reflecting the retiree's attitude to risk.

$$J\big(P(t), D(t)\big) = 1 - \exp\big(-\gamma[P(t) - P_B + D(t)]\big)$$

γ is the absolute risk parameter
P_B is a benchmark pension
$P(t)$ is the pension received at time t
$D(t)$ is the bequest amount

Personal discount rate and discount rate reflecting time preference are one and the same. All that they mean, in simple terms, is that the interest rate is a reflection of the premium on enjoyment nearer in time over more remote enjoyment.

The crucial part in the above analysis is that the rational agent or pensioner in this case is able to precisely describe his or her utility function. **Psychologists point out that individuals are not able to describe their risk preferences very clearly** and therefore their von-Neuman Morgenstern utility is unknown. Economists have countered this argument by suggesting that even if individuals cannot describe their utility functions, they behave according to rational utility functions. The solution is therefore to gauge an individual's level of risk aversion or shape of his or her utility function by a series of questions. However, in the psychology arena, the expected utility paradigm has come under fire with psychologists showing that risk and uncertainty questions are answered depending on how they are phrased! Thus hypothetical scenarios can be created in which axioms of utility are violated even though there is real money at stake.

Our goal is to help retirees decide on which annuity strategy to follow, without requiring much in the way of risk aversion parameters, inter-temporal rates of substitution, personal discount rates and elasticity of marginal utility. As already mentioned, these macroeconomic parameters are difficult, if not impossible, to measure accurately, and are founded on the existence of well-defined utility functions.

It was thus decided not to try and model personal utility functions, but stick to more objective risk and return measures.

We conclude this section with a statement from Roy (1952) who said, "In calling in a utility function to our aid, an appearance of generality is achieved at the cost of a loss of practical significance and applicability in our results. A man who seeks advice, about his actions will not be grateful for the suggestion that he maximise expected utility."

2.3.2 The risk value model

Risk value models are models used to evaluate decisions under risk. Preference for a gamble is determined by its riskiness and its value or worth. In a simple form of risk-value models, risk is measured by variance and value by expected returns. Sarin and Weber (1993) show that if risk and value are defined in an appropriate way, the rank order produced by the model can, but need not, be consistent with the expected utility approach of choice.

Dus, Maurer and Mitchell (2005) use a risk value model whereby "return" is the expected level of benefits as well as the expected possibility of bequest, and the "risk" is the possibility of not reaching a benchmark or desired level of consumption. Not only does risk here embrace possibility but also the severity of that shortfall if it does occur. Thus, put differently, risk is the expected shortfall. These are concepts that retirees can easily relate to.

In this paper we calculate the expected present value of benefits and probability of not reaching a specific benchmark at various points in the future, but do not consider the severity of shortfall or bequests.

2.3.3 The probability of ruin

There are a number of studies that have looked at the problem of ruin after retirement. Milevsky (1994) looks at a retiree who structures his or her living annuity portfolio in such a way that the probability of failing to meet a minimum required return is minimised. This minimum return is the return required to maintain a desired level of consumption. In Milevsky (1997), a probability of starvation is calculated. This is defined as the probability that one lives to the point where he or she runs out of money.

$$P(starvation) = \sum_{i=1}^{\infty} P^{\alpha}_{i/k} . P(N^* = i)$$

where N* is the time when wealth will be first zero, $P^{\alpha}_{i/k}$ is the probability of surviving to age $x + i/k$ where k is the number of compounding periods in a year.

Albrecht and Maurer (2002) also follow this methodology in their analysis of the German market.

What is noteworthy of the above measures is that they all consider the probability that wealth is zero before death; that is, a person outlives his wealth. In South Africa, the maximum living annuity drawdown percentage prevents someone from running out of wealth completely, so alternative "starvation" or ruin measures are required.

Zero wealth is a retiree's worst-case scenario; they may also be unsettled if their wealth falls to a level that threatens to plunge them into their worst-case scenario or jeopardizes their bequest intentions. Roy (1952) argues that investors think in terms of a minimum acceptable outcome, what he calls the "disaster level." The safety-first strategy is to choose the annuity strategy that has the smallest probability of going below the disaster level.

In our modelling we look at two disaster / ruin levels:

- **The income not being able to be maintained (i.e. the Rand value of monthly income falling to below the initial Rand income level)**
- **The income not being able to grow over time**

3 Modelling Procedure and Assumptions

In this section we detail the basis for the calculations in this paper. All analysis is based on a 60 year-old male retiring with R500,000 in capital on 31 December 2007. We project the monthly income he receives from the various annuity types until age 90.

3.1 Software Used

Modelling was predominantly done using Microsoft Excel.
Exceptions to this were the generation of the projected asset returns (which were created using specialist asset modelling software) and the with-profits annuity bonus rates (which were produced using MoSes actuarial modelling software).

3.2 Asset Projection Model

Future interest rates and investment returns were produced using a real-world (as opposed to risk-neutral) economic scenario generator. Consistent with current market best practice, the software and calibrations required for this were provided by a specialist external economic scenario provider (Barrie & Hibbert in this case).

3.2.1 Interest Rate Model

One of the reasons a specialist provider was chosen (as opposed to using more simple random walk modelling) was that a “proper” interest rate model was considered important when comparing different annuity types at different times in the future.

For these purposes, the Two Factor Black Karasinski interest rate model was used.
In this model:

- Short-term interest rates are assumed to be lognormal.
- Future interest rates are path dependent.
- The interest rate process is made up of two stochastic elements
- Future interest rates are always positive.

In this way the interest rate process should be economically consistent (the average return on cash should reflect the shape of the initial yield curve), and exhibit some volatility.

The bond yield curve as at 31 December 2007 was used as the starting point for the projections. Returns were projected monthly for 30 years, and 1000 return scenarios were generated.

3.2.2 Bond Returns

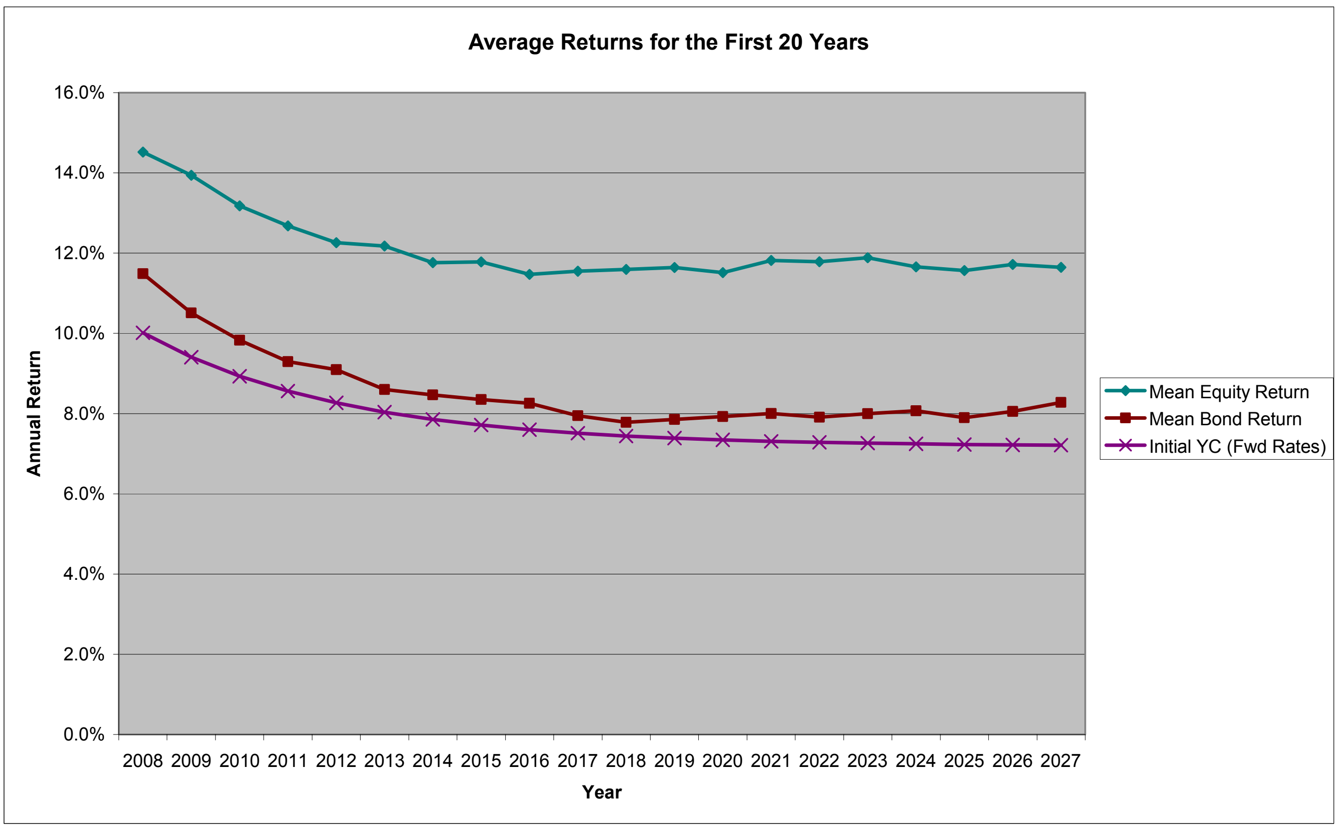
Yield curves get derived at each future time-step from the projected short-term interest rates. From this local bond returns can be calculated. For these purposes it is assumed a portfolio of 20-year bonds, with coupon rates reset to par at each time step, is always held.

An important point is that because we don’t assume a buy-and-hold bond strategy, one doesn’t earn the fixed initial yield to maturity of the bonds. Because of the downward sloping shape of the initial yield curve (the SA bond curve as at 31 December 2007), interest rates are expected to fall and one is expected to make bond capital gains. One then purchases another 20-year par bond, this time at a lower yield. A bond return in excess of cash returns thus emerges as the yield curve continues to fall.

3.2.3 Risky Asset Class Returns

Returns on "risky" asset classes (i.e. assets other than local cash and local bonds) are modelled as lognormal risk premiums in addition to the returns on local cash. The modelling here only uses bond and SA equity returns, and for SA equities an annual arithmetic mean risk premium (above local cash) of 4.5% was assumed.

It is important to note that because asset returns are based on a path-dependent interest-rate model derived from the initial yield curve, mean asset class returns differ for each year of the projection period. These mean returns (arithmetic average over the 1000 simulations) are shown in the graph below.



One can clearly see the consistent equity risk premium emerging, as well as bond returns averaging around 1% p.a. more than the yield curve's forward rates.

An interesting point to note is that the cumulative average returns over the 30-year projection period (i.e. accumulating the average returns shown on the graph) do not equal the average cumulative returns (i.e. the average of all 30-year returns across the 1000 simulations) over the period.

	Cumulative Average Return	Average Cumulative Return
Initial Yield Curve - Forward Rates	7.6% p.a.	7.6% p.a.
SA Bonds	8.5% p.a.	7.9% p.a.
SA Equity	12.0% p.a.	9.4% p.a.

In other words, although in any given year equities are expected to earn 4.4 - 4.5% more than risk-free, the average equity return over the period is only 1.8% above risk-free. The reason for this is volatility - a geometric average (e.g. compound return) is below an arithmetic average when the numbers being averaged are volatile. The authors have not investigated whether these cumulative risk premiums are appropriate or not.

3.2.4 Volatility and Correlation Assumptions

The model was calibrated using the following volatility and correlation assumptions:

Asset Class	Volatility p.a.	Correlation with Equity	Correlation with Bonds	Correlation with Cash
SA Equity	22%	100%	33%	5%
SA Bonds	10%	33%	100%	11%
SA Cash	1%	5%	11%	100%

3.2.5 Using Stochastically Generated Returns

Although the comment was made that is currently best practice in the market to outsource the asset model provision to a specialist provider, it is critical for those using the returns to understand their derivation (at least at a high level) and the implications thereof.

There will always be some subjectivity involved in the calibration (especially when it comes to a real-world model), and the extent of this needs to be understood.

3.3 Mortality Basis

The mortality tables used were the recently published South African Annuitant Standard Mortality Tables (SAIML98 and SAIFL98). These are the first standard mortality tables for South African immediate annuitants, and are based on industry data collected from 1996 to 2000. The mortality rates are consistent with the widely used PA(90) tables less an age adjustment (3 years for males, 2 years for females).

As recommended by Dorrington and Tootla (2007), allowance was made for mortality improvements by applying a reduction of one year of age for every twenty years projected.
The rates are shown in Appendix A.

3.4 Guaranteed Life Annuity Basis

The guaranteed life annuity calculation is in itself quite simple – one only requires the yield curve, a mortality basis and an expense / profit charge allowance. In fact, if one knows what the market annuity rates are, a calculation basis isn't even needed (future income is predefined, and doesn't need to be calculated).

The problem comes when trying to generate future guaranteed life annuity rates (for investigating the benefits of switching later in life). For this one does need a calculation basis, of which the most difficult aspect to ascertain is the expense / profit charge allowance. [4]

3.4.1 Replicating Market Annuity Rates - Deriving an Implied Reduction in Yield

To try and come up with a basis that represents reality as closely as possible, actual market annuity rates were compared to those implied by the yield curve. An implied "reduction in yield" (RIY) was calculated, representing the percentage per annum deduction from the yield curve required to provide the market annuity rates.

This RIY % can be thought of as the life annuity's expense and profit charge allowance, as well as any risk margin the life company has allowed for in the mortality rates (i.e. by pricing using lighter mortality or greater mortality improvements than best estimate).

The RIY calculation was done at the end of December 2007 (the starting point this research is based on) as well as at the end of June 2008. Two sets of information were used to confirm whether the RIY % derived was stable and accurate.

Market annuity rates were obtained from the Personal Finance newspaper and magazine, and were averaged across all the companies represented (as shown in Appendix B).

In calculating RIY %'s implied by the yield curve:

- No initial commission was used (it is not necessary to compare annuity types that all pay the same initial commission). This means that to compare rates derived to market rates (which are inclusive of maximum initial commission of 1.71% including VAT), derived rates were expressed per R98,290 consideration, comparable to the after commission "investable amount" of the market rates.
- Both BESA bond and swap curves were tested. The real world asset model uses bond returns, but some companies might be basing their rates off the swap curve (which has a slightly different shape), and that might show more stable RIY %'s.
- The yield curve on the Thursday of the week prior to the published market rates was used. The reason for this is that market rates are usually published the Friday before the week begins.

[4] The expense and profit charges that life companies price into their annuity rates aren't disclosed (and they needn't be – the potential annuitant is only interested in the net income, which he can compare across life companies).

The results of this analysis showed that:

- There was no significant stability advantage for either the bond or swap curve from one period to another (although the bond RIY is significantly lower).
- The RIY from bond curve seems to decrease with age, but this is not shown with the swap curve .
- Age 60 at 27 June seems to be an outlier (it has very high RIY). This was supported by checking RIY the following week, which was very similar to the RIY at 3 January. The authors were not able to ascertain the reasons for this. It may be due to an insufficient number of companies in the data set, or perhaps the yield curve lag is greater than 1 week (and the yield curve increased sharply over the period). This, however, doesn't really explain why older ages at 27 June seem reasonable.
- Either way, the average RIY % across all dates and ages only drops from 0.62% p.a. to 0.57% p.a. without the inclusion of this data point.

Refer to Appendix B for full information.

Overall a RIY of 0.6% p.a. was decided on.

Does this seem reasonable?
A pure fixed interest / bond unit trust fund might actually have a slightly higher management fee. One is thus led to ask whether the 0.6% isn't a little low - there doesn't seem to be much allowance for a mortality risk margin?
There are, however, two major factor countering this:

- Life companies may be pricing off the swap curve, where the RIY was 1.26% p.a.
- Life companies may be assuming investment strategies that yield returns in excess of risk-free, and allowing for this "yield pickup" in their pricing

Given this, it was thus concluded that using a RIY % of 0.6% p.a. relative to the bond curve probably doesn't seem unreasonable.

As an additional comparison, the money's worth ratio (defined as the ratio of market annuity rates to annuity rates implied by the yield curve) was calculated. To make it comparable to the work done by Rusconi (2006a), who unfortunately didn't have the benefit of the SA Annuitant Mortality Tables, initial commission was included. An average money weighted ratio of 94.5% for the 60 year old male was obtained, supporting Rusconi's findings that value for money is reasonably good.

3.4.2 Annuity Rate and Income Calculation

Once a RIY % was determined, projected annuity rates could be calculated. A starting retirement age of 60 was used as the base for the research, and a starting date of 31 December 2007.

This was done for each return scenario by present valuing unit cashflows at that scenario's bond curve (less the RIY) and under the assumed mortality rates.

A 10-year guaranteed term (consistent with the market information on which the RIY % was calculated) was assumed. This effectively means a term certain annuity for the first

10 years and for life thereafter. This may seem contradictory to the policy of maximising income over the annuitant's lifetime, but the authors felt that most annuitants were likely to add this feature and not including it would overstate the guaranteed life annuity income. Another reason for including this is that the market annuity rates that were calibrated to include a 10-year guaranteed term.

Note that the rates are projected into the future, based on the bond curve at that time.
The age of the annuitant was incremented with the projection i.e. the annuity rate in 20 years' time applies to an 80 year-old (compared to the starting age of 60). These rates aren't relevant when comparing the different annuity types at retirement (the 60 year-old gets the life annuity rate calculated at the start for the rest of his life), but are needed when looking at whether the annuitant should switch between annuity types over time. Mortality improvements were allowed for accordingly.

The investment amount / annuity contribution was then divided by the annuity rate to get the monthly income payable to the annuitant.

For a R500,000 investment the starting bond curve provides an annuity income to a 60 year-old male of R4,221 p.m. in arrear for life.

In addition, the annuity rate for a 5% p.a. increasing guaranteed life annuity was also calculated (although at inception only, not in future years).
For the 5% p.a. increasing pattern an initial income of R2,749 p.m. was obtained.

It is worth noting that an inflation-linked annuity was not examined. Although theoretically ideal for a retiree in terms of providing an income stream that maintains its purchasing power over time [5], this added significantly complexity and, due to limited supply of inflation-linked bonds, current levels of initial income are low compared to other annuity types (making it an unlikely choice).

[5] Although it can be questioned whether the inflation index used matches pensioner inflation

3.5 Investment Linked Living Annuity Basis

Unlike the relatively simple nature of the guaranteed life annuity, the living annuity has different variables (in the form of options to the investor) that need to be considered:

- The asset allocation (and how this varies in different scenarios), and
- The drawdown rate (and how this varies in different scenarios)

Different levels of each of these were tested using the projected asset returns and assuming a R500,000 initial investment amount.

3.5.1 Asset Allocations

Only two asset classes were used - local equities and local bonds (with mean return, volatility and correlation assumptions as per section 3.2).
The following asset allocations were modelled:

- **100% equity**
- **75% equity, 25% bond**
- **50% equity, 50% bond**
- **25% equity, 75% bond**
- **100% bond**

These allocations give us a range of portfolios, from very aggressive to very conservative, which we can then generate income on and compare to the other annuity types. Although a 100% equity portfolio doesn't comply with prudential investment guidelines (and thus shouldn't be chosen), the authors wanted to see how its risk and return features compared to the other allocations and annuity types. Of particular interest is also to analyse:

- The 50% equity portfolio relative to the with-profits annuity
- The 100% bond portfolio relative to the guaranteed life annuity

It was assumed the portfolios are rebalanced monthly.

As a future extension, we believe it will be worthwhile examining some of the newer absolute return or dynamically hedged portfolios that have a more asymmetric return profile (limited downside volatility).

3.5.2 Drawdown Rates and Income Strategies

Legally annuitants now have to draw down between 2.5% and 17.5% of fund value each year. This percentage is set at the beginning of the year, and can be revised annually.

Seven drawdown rates were tested, starting from the minimum 2.5% through to the maximum 17.5% in increments of 2.5%.
Putting these in context, for an initial investment of R500,000 the first year's income is:

- **R1,042 p.m. for the 2.5% drawdown rate**
- **R3,125 p.m. for the 7.5% drawdown rate (just above the 5% increasing annuity)**
- **R4,167 p.m. for the 10% drawdown rate (similar to the level life annuity)**
- **R7,292 p.m. for the 17.5% drawdown rate**

An important consideration is how the retiree chooses to change the drawdown rate over time. In good return scenarios (where capital is growing), it isn't necessarily an issue keeping the same % drawdown (the income will then grow accordingly).

The poor return scenarios are less obvious – does the annuitant:
- Keep the same drawdown %?
- Increase the drawdown % (subject to the maximum allowable) to maintain the Rand amount of his monthly income or to grow his income to meet cost of living increases?
- Decrease the drawdown % (subject to the minimum allowable) to try and maintain his capital?

Overseas research on this topic hasn't necessarily had to deal with drawdown limit complexity. Researchers often assume the same level of income throughout, and allow capital to reduce to zero.

Alternative strategies that have been suggested are drawing down over one's remaining life expectancy ("1/E[T]" method) and drawing down to a fixed age, for example the maximum death age ("1/T" method).

Another option sometimes used in the discretionary investment income space is to only receive the portfolio's income distributions as income, and never disinvest any capital. Income could then be stable and even grow over time (for equity dividends and property rental income). The capital value will fluctuate, but if the investor holds the assets indefinitely it doesn't matter (apart from having to ensure the legal drawdown limits aren't breached). The income received is treated as a stand-alone income stream (which for most assets it is), rather than a percentage of capital. Capital losses / gains are never crystallised. This is similar in concept to the 3rd bullet point above.

After evaluating all of the above, it was decided to model three drawdown strategies:
Strategy 1: Keep the same drawdown % throughout
Strategy 2: Adjust the drawdown % each year (up or down) to maintain the Rand amount of income
Strategy 3: Adjust the drawdown % each year to grow the Rand income by 5% p.a.

Strategy 3 was specifically chosen with reference to the new living annuity disclosures and to compare to the 5% increasing guaranteed annuity.
In strategies 2 and 3 the drawdown was at all times constrained by the minimum and maximum % limits.

Although not modelled, the authors feel the option to only receive the portfolio's income (and never disinvest capital) merits future consideration.

3.5.3 Capital Remaining on Death

Because the aim of this paper is to evaluate which annuity strategy generates maximum income over a retiree's lifetime, we do not (apart from in a few special cases) explicitly look at the capital value remaining on death.

This does not mean capital is not important. In terms of maximising income, one can hypothesize that:

- Because of the drawdown restriction, the retiree can never deplete the capital completely. This effectively limits his ability to maximise income over his lifetime.
- The lower the capital remaining at the end, the more income the retiree has managed to extract.

In other words, the higher the return earned and the higher the drawdown rate, the more the retiree will have received from the living annuity.

On the other hand, one needs to look at the probability of the annuity income not meeting the retiree's needs i.e. is the income profile acceptable (in terms of staying level or growing over time). This factor opposes the previous one – if capital depletes the income will eventually decrease, and then isn't likely to meet the retiree's needs.

We look at both of these factors for all combinations of asset allocation and drawdown:

1. The expected present value of total future income
2. The probability of not meeting a predefined level of income during a year ("ruin")

3.5.4 Charging Structures

The following charging basis was assumed:

- Asset management fees of 1.6% p.a. on the equity portion of the portfolio and 0.9% p.a. on the bond portion. (For comparison, fees on some of the large unit trust funds in the market are shown in Appendix C.)
- An ongoing advice / commission of 0.57% p.a. (0.5% + VAT), standard in the market
- To be consistent with the life annuity, no initial commission charge.
- No initial expense charge, but an ongoing expense charge of 0.25% p.a.

Based on this, the following total living annuity charges were used for each portfolio:

Allocation	**Total Fee Used**
100% equity	**2.40%**
75% equity, 25% bond	**2.25%**
50% equity, 50% bond	**2.10%**
25% equity, 75% bond	**1.90%**
100% bond	**1.70%**

Ad hoc checks on some of the newer "all-in" fee funds (where asset management, expense and advice charges are bundled together) show that these total fees are not out of line with the market. If anything, these are on the low side compared to some of the more exotic funds, especially those with various underlying layers (fund of funds).

3.6 With-Profits Annuity Basis

This is the most complex to model out of the three annuity types, not only because of the varying underlying components but also because there are relatively few product providers (particularly in the retail market). This means standardised, disclosed market practices aren't easy to come by.

The modelled product design was based on a traditional with-profits annuity, with information derived from the Old Mutual and Metropolitan Life websites, as well as discussions with the companies' respective product actuaries.

In exchange for his investment / annuity contribution, the annuitant receives a starting guaranteed level of income (below that paid by a guaranteed life annuity).
The life company then invests the annuity contribution, and based on the investment returns earned declares annual bonuses. These increase the annuity income, and become guaranteed (in traditional terms, they "vest"). The income can never decrease, and the retiree expects that it will grow over time to partly / fully meet cost of living increases.

The key factors to consider thus are:
- How the initial guaranteed income is calculated
- How the annuity contribution is invested, and what investment charges there are
- What the basis for bonus declarations is

3.6.1 The Initial Guaranteed Income

The initial income is determined by the post retirement interest rate (pri) / pricing interest rate used by the life company. This rate is disclosed to the annuitant (who may even be given the choice between different rates), and creates expectations for the level of future bonus.

The expected level of future bonuses is governed by the relationship below:

$$\text{Expected bonus } = \max\left(0, \left(\frac{1+R-M}{1+pri} - 1\right)\right)$$

where: *R = Expected return of the underlying portfolio*
M = charges deducted by the life company (e.g. investment and capital charges)
pri = pricing interest rate.

Roughly, therefore, $\textit{Expected bonus} \approx \textit{Return} - \textit{pri} - \textit{charges}$.[6]

From the above relationship we see that the higher the pri chosen, the smaller the expected future level of bonuses. In addition, the higher the pri the more constrained the investment strategy is likely to be, which further reduces the bonus expectation. Historically different levels of pri have been offered, depending on interest rates at the time.

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6 Further on we show how, in one return scenario, the bonus rate compares with the return less the pri

Prevalent in the market at the moment is a 3.5% pri

- The company is guaranteeing a minimum investment return of 3.5% p.a.
- If the portfolio is expected to deliver investment returns of *inflation + 3.5% p.a.* after charges, the annuitant can expect his income to grow (via bonus declarations) with inflation.

The 3.5% pri seems to be currently marketed as having bonus expectations of just below inflation, and is the one assumed here.

An annuity rate can then be calculated using this 3.5% interest rate and the annuitant's mortality, consistent with the guaranteed life annuity calculation described in section 3.4.

- Note that it is also assumed that the with-profits annuity offers a 10-year guaranteed term.

A difference between the with-profits annuity and the guaranteed life annuity is that expense charges seem to be explicitly disclosed, and deducted from the annuity consideration before the calculation of the initial guaranteed income.
Consistent with this, the following expense / administration charges were allowed for: [7]

- R500 initial expense charge
- 2% of annuity payment ongoing expense charge, capitalised upfront

This results in total expense charges of R10,000 being deducted from the R500,000 investment amount.

The net annuity consideration of R490,000 was divided by the (high) annuity rate to get the initial guaranteed monthly income payable to the annuitant.
The resulting initial guaranteed income for a 60 year-old male is R2,898 p.m. [8]

It can be seen that this is well below the level guaranteed life annuity's R4,221 p.m., but very similar to the 5% increasing annuity's R2,749 p.m.

3.6.2 The Investment Strategy

The life company needs to determine what asset allocation to employ to be able to provide the initial guaranteed income as well as reasonable future bonus rates.

Investing in too aggressive a portfolio will expose the company to too great a risk of not being able to meet the guaranteed income.
At the opposite end of the scale, the company could fully match the initial guaranteed income using a bond / swap portfolio (as with a guaranteed life annuity) but would then be left with very little investment freedom (a very low possible equity allocation and low bonus expectations).

- This is actually far too conservative - the unmatched portion will never reduce completely to zero, meaning (as strange as this sounds) one does not need to fully match the initial guaranteed income to be fully matched.

[7] Based on Old Mutual's Platinum Pension 2003 disclosure report of March 2008.
[8] Old Mutual quoted a rate of R2,911 p.m. and Metropolitan quoted one of R2,907 p.m.

Historically the life companies have adopted a strategy somewhere in between – not matching the guaranteed income fully, but matching a proportion of it. The life company carries the residual risk that the rest of the portfolio cannot provide the rest of the guaranteed income. There is then typically a capital charge to cover this residual risk to the insurer.

Companies are increasingly making use of advanced capital modelling / dynamic hedging techniques to try and determine optimal risk-return trade-offs, which changes the above situation. Simplifying things somewhat, modern hedging techniques / instruments can create a fully matched position without the excess conservatism described above.

It must not be forgotten that the aim of this paper is merely to compare income from different annuity types, not to derive an optimal asset allocation / risk management strategy for a with-profits annuity. The approach adopted was thus to allow for some matching (and thereby partly lock into interest rates upfront) and try and ensure the overall asset allocation and expected future bonus rates are consistent with those offered in the market. If anything, more modern matching / allocation techniques should improve the with-profits annuity proposition over that modelled in this paper.

The assumption was made that the insurer matches 70% of the guaranteed initial income.

The amount to be invested in the matched portion was then derived as follows:

- The 31 December 2007 bond yield curve was used to calculate annuity rates (again according to the calculation method in section 3.4)
- These were multiplied by 70% of the guaranteed initial income to determine the amount required to be invested in the matched portfolio.

This gave an amount of R230,563 per R490,000 annuity consideration, or an overall matched proportion of 47%.

Examining Old Mutual's Platinum Pension 2003 disclosure report of March 2008, one sees that 42% of the portfolio backing their 3.5% pri option is invested in matching bonds.
This suggests our strategy may be too conservative, although we must remember that the life company's actual portfolio will be made up of investments that have come in at a range of different yield curve levels, not just the 31 December 2007 yield curve.

Now, turning to the unmatched portion. Again the Old Mutual report is used to see that almost all of it is in risky asset classes. To be consistent with the manner in which the living annuity was modelled, it was assumed that the with-profits annuity portfolio is also invested only in bonds and domestic equities. However, investing nearly 60% in unmatched assets, domestic equities in our case, was considered risky as there wasn't the privilege of holding diversified assets like local property, overseas equities, bonds and cash due to the fact that we did not model these asset classes. Thus, to achieve the diversification benefit, it was felt that a lower proportion should be held in domestic equities. We settled for 50%, and we could thus retain our initial assumptions.

We thus had the following asset allocation:

- **50% "Matched" assets (i.e. a guaranteed life annuity, with yields locked into upfront)**
- **50% "Unmatched" domestic equity assets**

This type of "balanced" portfolio could be expected to earn in the inflation + 3.5% range, hence the expectation of near inflation bonuses.

Note though the important differences between this and a living annuity with the same asset allocation:

1. The fixed interest portion is matched, meaning the yields are locked in upfront (and are not – assuming mortality experience is as expected - subject to capital value fluctuations during the term)
2. Expected mortality is built into the guaranteed income. All other things being equal, the with-profits annuity should provide a higher income than a living annuity because the investment capital in the with-profits annuity is being used up in providing the income (the mortality credits argument from section 2.2.1).

3.6.3 Charges

Again referring to the Old Mutual disclosure report, one sees that their with-profits annuity carries:

- A capital charge of 1% p.a.
- An investment management fee of between 0.7% and 0.75% p.a.
  In the model, an investment fee of 0.75% was used. It makes sense that this 0.75%:
  - Is higher than the RIY on the guaranteed life annuity
  - Is lower than the fee charged on the equity portfolio in the living annuity (which is a retail product, compared to the with-profits annuity which is primarily an institutional product)

Metropolitan quotes a slightly lower total (capital + management) charge of 1.45% p.a. for their traditional 3.5% pri option.

Our modelling assumed a total average charge of 1.75% p.a., which was deducted from the with-profit annuity returns.

3.6.4 Bonus Declarations and Smoothing

Now that an asset allocation and charge level has been established, one needs to determine how the bonuses are calculated.

Boulle and Maitland (2006) discussed in their paper the various factors that influence the way in which generated investment returns get declared as bonuses. They showed that the main forces that drive this process are policyholder and market requirements and demands of shareholders and the life company. Any bonuses declared were thus a balancing act between

- Reflecting investment and market conditions
- Maintaining real bonus rates
- Smoothing between bonus declarations
- Reaching a full distribution of investment income and capital, net of margins, over time
- Maintaining solvency and profitability demands of shareholders and the life company, and
- Minimising cross subsidies between members

The bonus philosophy developed for the with-profit annuity model drew upon the above factors. In other words, it attempted to take into account market conditions and allow for bonuses declared to be revised downwards if they threatened the solvency of the product. Thus in developing a formal bonus philosophy the following factors were taken into account:

- ***The five-year geometric average of past returns***. This was used as an indicator of the potential bonus (the "bonus indicator"), and is the first stage of the smoothing process.
  Needleman and Roff (1995) show that there are numerous methods of directly smoothing the achieved investment return. They categorise these as:
  - Methods which smooth historic returns in a deterministic manner; and
  - Methods that factor in the actuary's view as to the long term expected returns.

  The first approach, which is the method we adopt in this paper, has the advantage of complete objectivity, whereas the second approach allows for an element of judgement. The latter may be considered an advantage if the choice of the future return is consistent with that adopted for other purposes and the impact of different future returns is fully tested.

- ***The solvency of the product***. The bonus smoothing reserve was used as a proxy for this.

- ***Bonus smoothing reserve (BSR)***. This was calculated as the difference between the net value of assets underlying the with-profits annuity fund minus the discounted value of the guaranteed income. The guaranteed income here includes actual bonuses declared and the expected bonus, now as measured by the bonus indicator. A positive BSR for example represents the amount the with-profits annuity has over and above meeting the potential bonus (reflected by the bonus indicator).

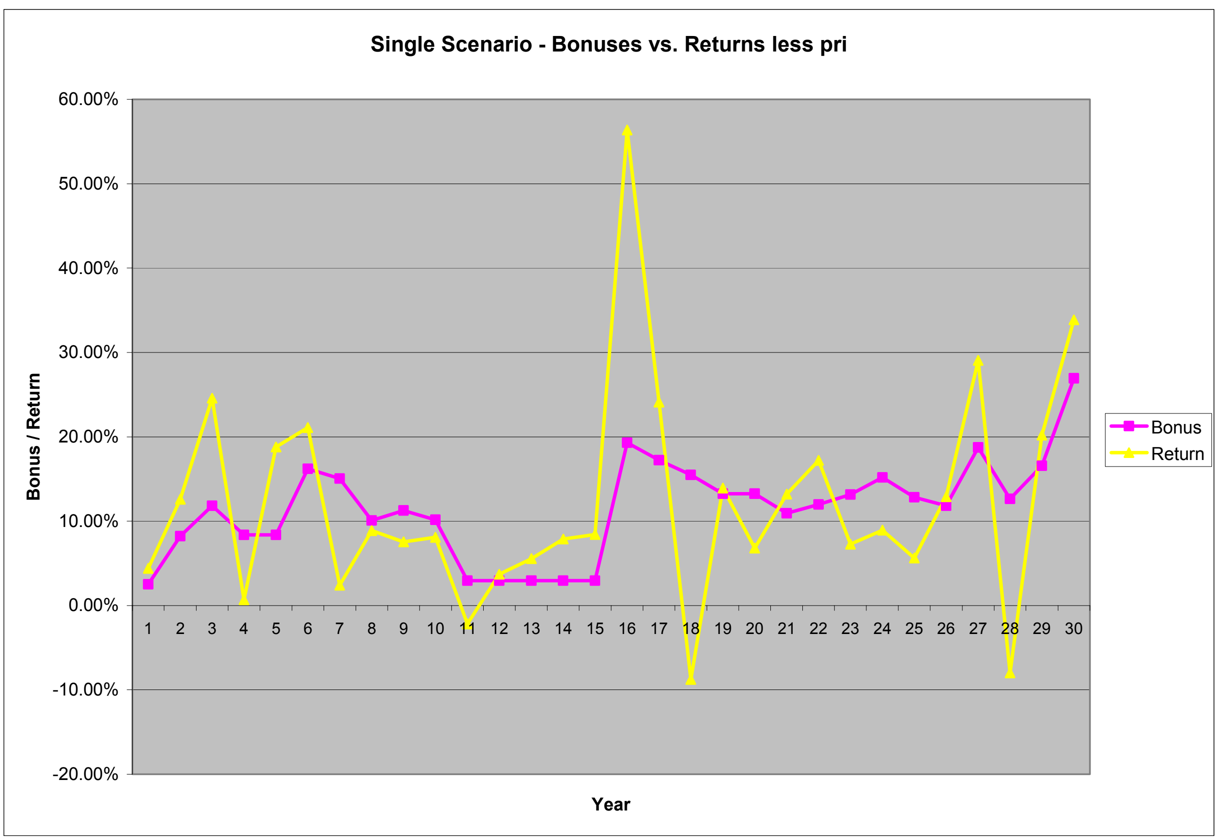
The bonus indicator was revised up or down depending on the level of the BSR. For example, if the bonus indicator resulted in the BSR being more than 5%, the excess above 5% was declared away (albeit by spreading it over 4 years).

- ***The initial BSR***. The initial BSR was set equal to zero. The modelled annuitant was thus not entering an existing fund with either a healthy or unhealthy smoothing reserve. The bonuses declared in the first 5 years were however constrained in order to allow this reserve to grow. This was done by capping returns before these were passed on as bonuses and spreading any excess funds over the maximum BSR (i.e. 5%) over 4 years.

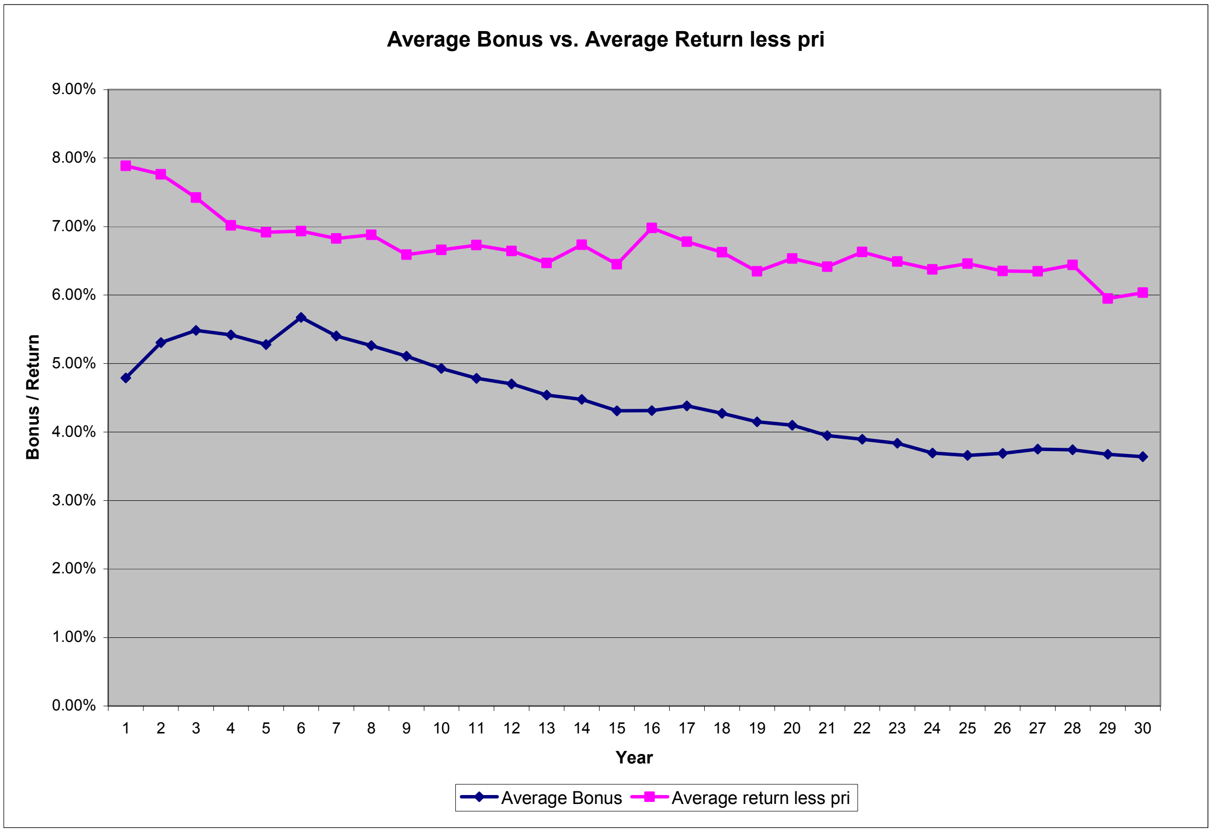
- ***Smoothing function for changes in bonus rate.*** The bonus rate was not allowed to move by more than one percentage point from the previous declared bonus, if the difference between the adjusted bonus indicator and the previous years' bonus was within the –1% and 1% band. In this case, the previous bonus was maintained or declared.

- ***Finally, the bonus was not allowed to be below zero.***

The graph below shows one simulation, comparing the bonus declared with the returns earned. One can see the smoothing effect at work:



The following graph shows the average bonus rate in each year (across all 1000 return scenarios) compared to the average return earned less the pri.



Here one can clearly see the effect of the investment charges on the bonus rates. The second reason for the gap between bonuses and returns less pri is the fact that some surplus gets held back. The average final BSR, i.e. the BSR at the end of the 30-year projection, was +2%.

The spike in bonuses seen at year 6 is a result of a more aggressive bonus distribution from then on. Surplus was held back in the first 5 years of the projection in order to build up the bonus smoothing reserve, which was assumed to be zero at the start of the projection.

4 Results

After setting the calculation basis and projecting the retiree's income, we can look at the risk and return characteristics of each annuity type. The following ways of assessing this were chosen:
- Risk as the likelihood of not being able to meet specific income needs (probability of ruin)
- Return as the expected present value of future income received
- An alternative measure of risk as the variability in the present value of income

4.1 Ruin Probabilities

The living and with-profits annuities both have an element of uncertainty to the future income they provide. One way of measuring this is by assessing in how many of the 1000 scenarios the income in a particular period falls below a certain benchmark.

We have defined two benchmarks to measure against:
1. The initial level of income
2. The initial level of income increased by 5% p.a.

The rationale for these is straightforward – either the retiree is looking to maintain the initial level of income or grow it over time.[9] If in any period his income falls to below the target level, his needs are not being met (= ruin).
This is not as extreme as Milevsky's "starvation" scenario, but as previously discussed the South African retiree can never actually totally run out of capital.
Another important reason for using these benchmarks is that the retiree could have received these income patterns from the guaranteed life annuities, and the new living annuity disclosures specifically refer to a 5% increasing guaranteed annuity.

Drawdown strategy 1 (same drawdown %) was tested against both benchmarks, whilst drawdown strategy 2 (maintaining the Rand amount of income) was specifically tested against the first benchmark. Similarly, drawdown strategy 3 (growing the Rand income by 5% p.a.) was tested against the second benchmark.

Complete results for living and with-profits annuities are shown in Appendix D, with the main findings addressed in this section.

4.1.1 Likelihood of Not Maintaining the Initial Rand Amount of Income

Naturally the lower the drawdown rate, the higher the likelihood of being to maintain the initial income.

[9] Although as mentioned in the introduction, individuals have a variety of different circumstances, needs and preferences, and these ruin definitions might not apply to a particular retiree.

Comparing the drawdown strategies

The key findings in comparing drawdown strategies 1 and 2 are:

- Irrespective of the asset allocation, for drawdown strategy 1 the retiree has a greater than 50% probability of not being able to maintain the initial income for longer than 5 years if his drawdown rate is above 5%.
- Drawdown strategy 2 reduces the ruin probability, with the retiree being able to draw up to 7.5%. It is only for the two most conservative asset allocations (100% bond and 25% equity) and the riskiest one (100% equity) that the ruin probability goes above 50% late in the term.
- This makes sense – drawdown strategy 2 is designed around providing a level income, and should have lower ruin probabilities.

Comparison of Ruin Probabilities for Drawdown Strategies 1 and 2							
50% Equity	**End of Year**						
7.5% Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
Drawdown Strategy 1	0%	45%	**51%**	**57%**	**60%**	**57%**	**55%**
Drawdown Strategy 2	0%	0%	9%	22%	35%	44%	47%

- Note how the ruin probability for drawdown strategy 1 is often high early on. The reason for this is that income is a fixed % of capital, meaning any scenarios with initial negative returns will have capital values that reduce, and corresponding income falls. These are not necessarily poor long-term scenarios – over time the expected returns come through and the ruin probability falls.
- One needs to remember that portfolio returns are the same between the two strategies, meaning that if drawdown strategy 2's ruin probability is lower it is purely because it is extracting more capital from the annuity portfolio. This isn't sustainable if the drawdown rate is too high (the capital will start to reduce and eventually the 17.5% cap will be reached and income will then also reduce), and the ruin probability eventually catches up to or even exceeds drawdown strategy 1's.

From this, it appears one can manage the drawdown rate to maintain one's Rand income as long as the initial income is not above 7.5% of capital.

The 7.5% drawdown limit makes sense. Section 3.2.3 showed us the expected risk-free rate was very close to this (at 7.6% p.a.). In order to sustain a 10% drawdown rate[10], the underlying portfolio would need to earn a gross (before asset management, advice and expense charges) return between 11.7% and 12.4% (for the 100% bond and 100% equity portfolio respectively). This cannot be expected to happen (we expect returns in the region of 8.5% p.a. for the bond portfolio and 12% for the equity portfolio), hence the high ruin probabilities for a 10% drawdown rate.

[10] Equivalent to the level guaranteed life annuity's income

Comparing the asset allocations

In term of how the different asset allocations compare:

- For the lowest drawdown rates (2.5% and 5%), the higher the equity exposure the higher the ruin probability. This makes sense – due to their low volatility the conservative portfolios have virtually no chance of ruin, whereas the higher equity portfolios do have some chance.
- For the higher drawdown rates (10% and above), however, the riskier portfolios actually have lower ruin probabilities. The reason for this is there are times at which the high equity portfolios will deliver high enough returns to support a high drawdown rate (whereas the bond based portfolios have little or no chance of doing so). The downside to this is that although this could happen, it cannot be expected to happen – the ruin probability is still very high.

The "break-even" 7.5% drawdown rate is more interesting and less clearcut. Looking at the effect of different asset allocations for each drawdown strategy:

Comparison of Ruin Probabilities for Different Asset Allocations – 7.5% Drawdown Rate							
Drawdown Strategy 1	**End of Year**						
	Start	**5**	**10**	**15**	**20**	**25**	**30**
100% Bond	0%	49%	**69%**	**87%**	**90%**	**86%**	**85%**
25% Equity	0%	45%	**57%**	**66%**	**72%**	**68%**	**67%**
50% Equity	0%	45%	**51%**	**57%**	**60%**	**57%**	**55%**
75% Equity	0%	47%	**51%**	**55%**	**57%**	**55%**	**54%**
100% Equity	0%	50%	**55%**	**56%**	**58%**	**56%**	**56%**
Drawdown Strategy 2	**End of Year**						
	Start	**5**	**10**	**15**	**20**	**25**	**30**
100% Bond	0%	0%	1%	7%	34%	**79%**	**97%**
25% Equity	0%	0%	1%	11%	26%	42%	**51%**
50% Equity	0%	0%	9%	22%	35%	44%	47%
75% Equity	0%	3%	18%	32%	41%	47%	50%
100% Equity	0%	9%	26%	39%	48%	**52%**	**54%**

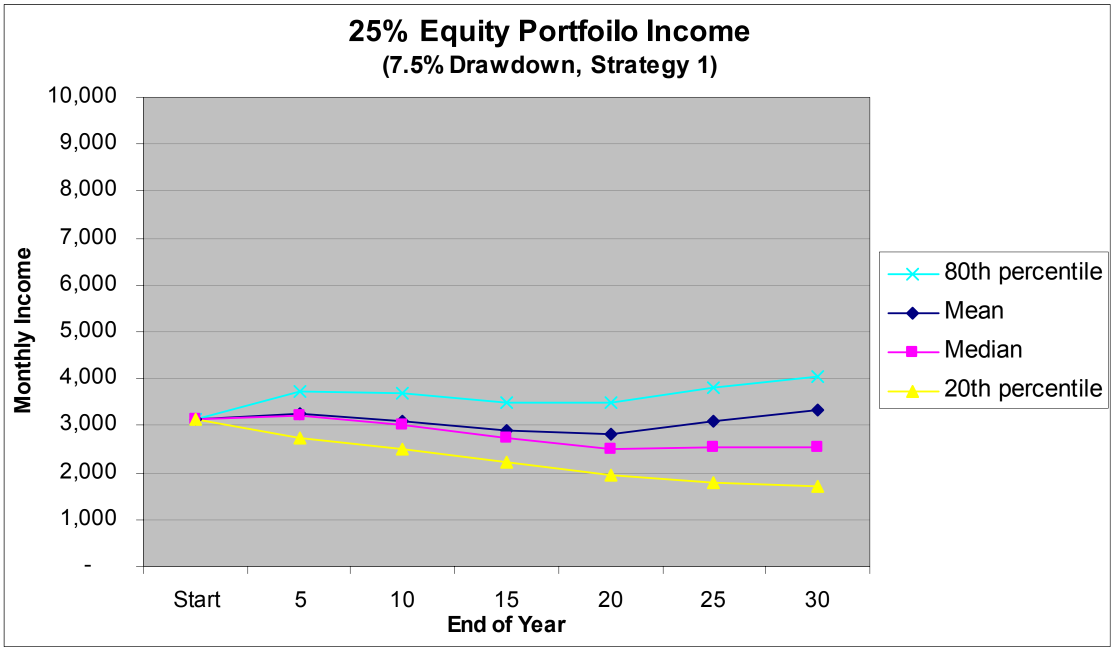
Drawdown strategy 1 doesn't show any clear winner. If anything, the higher equity portfolios are better as one moves further into the term.

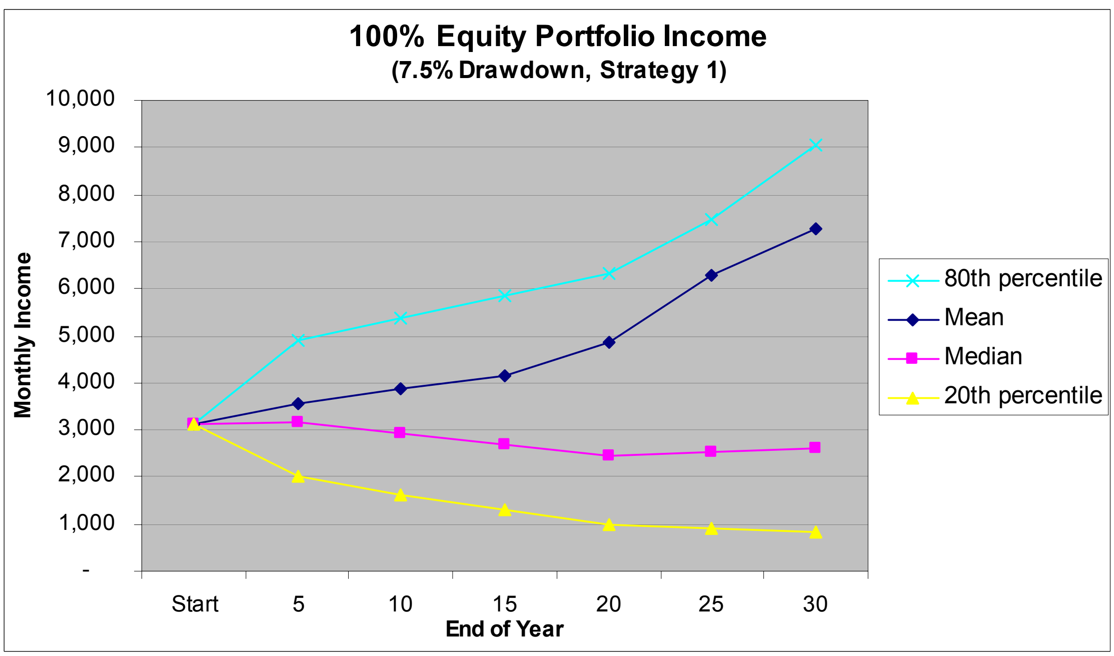
Drawdown strategy 2 exhibits more of a pattern:

- The 100% bond portfolio has the lowest ruin probability up until the end of year 15
- The 25% equity portfolio has the lowest ruin probability at the end of years 20 and 25
- The 50% equity portfolio has the lowest ruin probability at the end of year 30 (marginally better than the other portfolios with equity exposure, with the bond portfolio now having a very high ruin probability).

One might simplistically reason that the 100% equity portfolio should have the highest expected return, and thereby the highest chance of being able to provide the 7.5% drawdown rate. This is partly true, but gets offset by the fact we are not just dealing with the average / mean scenario – we are looking at the number of scenarios in which the income cannot be maintained. The higher volatility of the 100% equity portfolio increases these ruin probabilities.

One also needs to distinguish between the mean and median level of income. Looking at how the mean, median and 20th and 80th monthly income percentiles compare for the 25% and 100% equity portfolios (drawdown strategy 1):



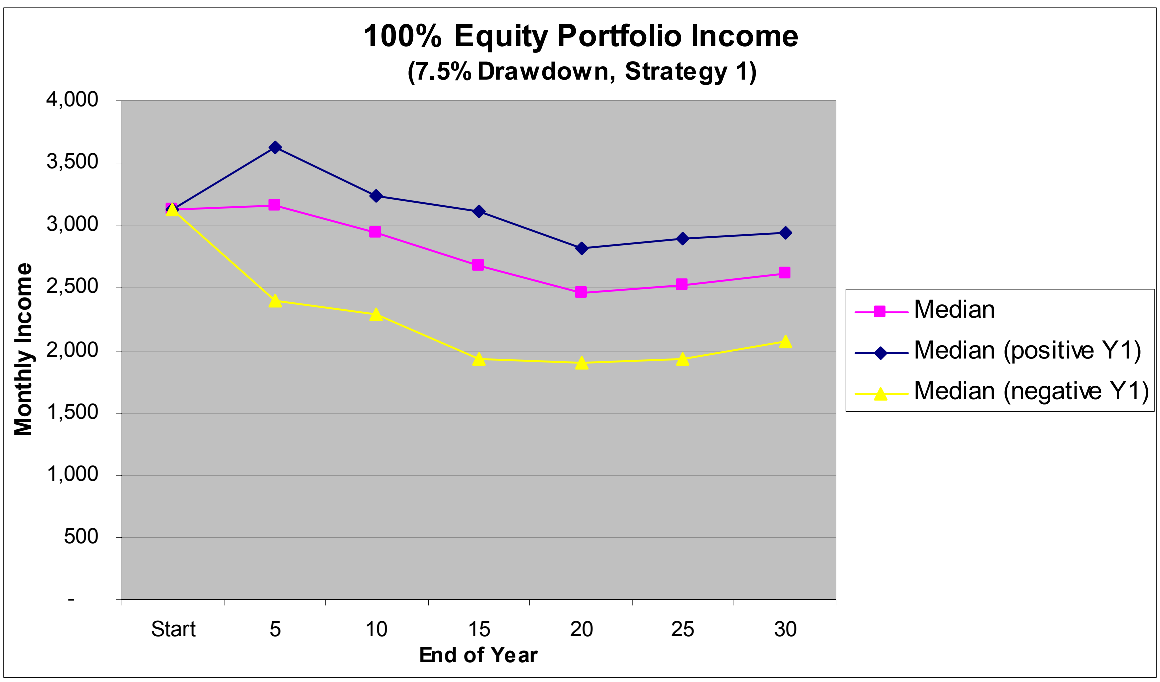


One can clearly see the wider income bands of the more volatile 100% equity portfolio. The key point is that although it provides a much higher mean / average monthly income than the 25% equity portfolio, its median monthly income is similar.

One reason the 100% equity portfolio's mean is above its median is that the asset returns are not symmetrical (they are derived from lognormally distributed risk-free rates). A far more significant reason is that drawing down a fixed % of the capital creates a very skewed income distribution. In very poor equity return scenarios where the capital depletes, the Rand amount of the following year's income also reduces. The capital can never deplete fully, effectively dampening the downside of the income.

This, however, still doesn't explain why the 100% equity portfolio's median isn't above the 25% equity portfolio's median. This warrants further analysis, but the authors believe the main reason for this is volatility, and resulting **path dependence of income**. In poor return scenarios where capital reduces, the income disinvested further reduces the capital. This means that a higher future return is required to recover to the original capital position. Another way of thinking of this is as "negative Rand cost averaging" – one is disinvesting in a falling market, reducing the ability to benefit from any subsequent market upturns.

This path dependence can be seen in the graph below, which shows the median income from those scenarios with a positive first year return compared to those with a negative first year return:



One can see that a negative first year return dramatically reduces the median income.
A more conservative portfolio will have fewer and less severe negative return scenarios, reducing the effect of this path dependence.

Now, returning to the ruin probabilities for the different asset allocations. As already discussed, drawdown strategy 2 is clearly superior in minimising the likelihood of not maintaining one's original income. But which asset allocation is optimal?

One way of assessing this is to bring mortality into the equation, multiplying the ruin probabilities by the likelihood that the annuitant survives. This gives the following "mortality adjusted" ruin table:

Mortality Adjusted Ruin Probabilities – 7.5% Drawdown Rate							
Drawdown Strategy 2	**End of Year**						
	Start	**5**	**10**	**15**	**20**	**25**	**30**
tPx [11]	1.00	0.91	0.79	0.65	0.49	0.32	0.17
100% Bond	0%	0%	1%	5%	17%	26%	16%
25% Equity	0%	0%	1%	7%	13%	14%	8%
50% Equity	0%	0%	7%	14%	17%	14%	8%
75% Equity	0%	3%	14%	21%	20%	15%	8%
100% Equity	0%	9%	21%	25%	24%	17%	9%

It must be noted that the authors have some discomfort doing this. Overlaying the likelihood of survival masks the underlying ruin probabilities (that will be experienced by those living annuity investors who actually survive). For example, the 32% of retirees that are expected to make it to age 85 actually have a 79% chance of not being able to maintain their initial income. **This illustrates the mortality risk that the living annuity investor takes on.** Having said this, we have already pre-selected the above combinations as being potentially acceptable, and are just using this technique to identify the one expected to be optimal.

From this it appears the 25% equity portfolio is optimal (having the lowest ruin probability in all cells except 15 years, where it is still low). Even without taking survival likelihood into account, this portfolio only has a 26% chance of ruin in the first 20 years. Even this may (will) be too high for some retirees.

For completeness, to finish this section we look at the main reasons why investing one's living annuity in a 100% bond portfolio cannot maintain a Rand income starting at 10% of capital (i.e. compared to why the guaranteed life annuity can). These are:

1. The living annuity doesn't efficiently use up capital over the term (the retiree doesn't benefit from the mortality credits over his life expectancy).
2. The bond returns are not locked into upfront – there is some future volatility in them, causing some probability of ruin.

[11] Using the SAIL98 Mortality Table for a 60 year old male

Summarising the findings of this section:

- **One can manage the living annuity drawdown rate to maintain one's Rand income as long as the initial income is not above 7.5% of capital.**
- **In doing so, and taking one's likelihood of survival into account, a 25% equity portfolio appears to be the optimal asset allocation.**

This doesn't compare favourably to:

- **The level guaranteed life annuity, which has zero ruin probability and equates to a 10% initial drawdown rate.**
- **The 5% increasing guaranteed life annuity or with-profits annuity, which have zero ruin probabilities (in fact, provide increasing income streams) and equate to a 7.5% initial drawdown rate.**

4.1.2 Likelihood of Not Achieving a 5% Increasing Income

As for the level income, the probability of not being able to grow the initial income by 5% p.a. was assessed. The rationale for this was that ideally retirees should purchase a retirement income stream that grows over time to counter the effects of inflation. One could theoretically have modelled future inflation stochastically and measured the ruin probability relative to this, but:

- The new living annuity drawdown disclosures and the 5% increasing annuity [12] could more clearly be assessed relative to a 5% p.a. increasing benchmark
- As at December 2007, 5% was not that far off future expected inflation (with an expected future average risk-free rate of 7.6%)

It is important to note that the with-profits annuity is likely to provide better inflation protection in a high inflation environment than the guaranteed annuities are. This could be important in South Africa.

Ruin probabilities for all living annuity drawdown rates, asset allocations and drawdown strategies 1 and 3 as well as for the with-profits annuity are shown in Appendix D.

As with maintaining a level income, the living annuity drawdown strategy customised to providing a 5% increasing income (drawdown strategy 3) exhibits lower ruin probabilities for low drawdown rates. What is less clear is whether a 2.5% or 5% initial income is acceptable:

Comparison of Ruin Probabilities - Drawdown Strategy 3							
2.5% Initial Drawdown	**End of Year**						
	Start	**5**	**10**	**15**	**20**	**25**	**30**
100% Bond	0%	0%	0%	0%	0%	0%	0%
25% Equity	0%	0%	0%	0%	0%	1%	1%
50% Equity	0%	0%	0%	0%	2%	6%	11%
75% Equity	0%	0%	0%	3%	8%	15%	20%
100% Equity	0%	0%	2%	8%	18%	25%	30%
5% Initial Drawdown	**End of Year**						
	Start	**5**	**10**	**15**	**20**	**25**	**30**
100% Bond	0%	0%	0%	9%	**84%**	**99%**	**100%**
25% Equity	0%	0%	0%	12%	**53%**	**78%**	**88%**
50% Equity	0%	0%	5%	23%	48%	**63%**	**71%**
75% Equity	0%	0%	13%	33%	50%	**60%**	**67%**
100% Equity	0%	3%	20%	40%	**53%**	**62%**	**67%**

Irrespective of the asset allocation, the 2.5% initial drawdown has low ruin probabilities throughout. The 5% initial drawdown has relatively low ruin probabilities until about year 15 or 20, whereafter they increase significantly. Here one could again overlay the likelihood of surviving to each age, but the drawbacks with this have already been discussed.

[12] Although one could have used an inflation-linked guaranteed annuity, it was felt that the depressed yields and low initial income levels make it an unlikely choice (alternatively expressed, the retiree has to pay a high price for an inflation guarantee).

There is no clearcut optimal option here, and the choice will eventually depend on the risk-preference of the retiree. The authors would suggest that this be looked at in conjunction with the expected future income level from each option (examined next).

Here one also needs to look at the ruin probability of the with-profits annuity:

With-Profits Annuity	**End of Year**						
	Start	**5**	**10**	**15**	**20**	**25**	**30**
	0%	**50%**	**55%**	**57%**	**61%**	**63%**	**66%**

The odds are just less than even that the with-profits annuity's income will increase at 5% per year. This is not unexpected – the 3.5% pri option used is marketed as having bonus expectations at / just below inflation, which should be in the region of 5% here.

One needs to bear in mind that the with-profits annuity has an initial income level similar to a 7.5% initial living annuity drawdown. It is thus impressive that it has ruin probabilities similar to a living annuity with a 5% initial drawdown. This is combination of the mortality credits / capital usage effect, having locked into bond yields upfront with 50% of the portfolio as well as the smoothing of bonus rates.

From a risk point of view, however, these are all superseded by the 5% increasing annuity. The retiree is 100% certain of receiving an income that grows at 5% p.a., and the initial income level is similar to the with-profits annuity / 7.5% initial drawdown living annuity.

In summary – if the retiree is concerned about not being able to receive an income that grows at 5% p.a.:

- **The with-profits annuity provides the highest level of initial income, but the ruin probability is above 50%**
- **The 5% increasing guaranteed annuity provides a similar level of initial income with no ruin probability**
- **The living annuity can be managed to provide an increasing income stream (i.e. by employing drawdown strategy 3) at relatively low risk, but only if the initial drawdown rate is 2.5% or 5%.**

4.2 Present Value of Income

We now turn to the expected return from each annuity type.
Appendix E shows the expected present value of future income, as well as the 20th and 80th percentiles of the present value of income.

This present value of income was calculated by discounting the future income stream at the starting yield curve[13] and under expected mortality rates.
For the living and with-profits annuity, this was done for each of the 1000 return scenarios, and the resulting discounted income could then be averaged (for the EPV of income) and percentiles could be calculated.
It goes without saying that the percentiles are the same as the EPV for the guaranteed annuities.

The annuity strategies with the highest EPV are as follows:

Annuity Strategies with the Highest EPV of Income	**20%**	**EPV**	**80%**
With-Profits Annuity	386,736	**528,652**	630,483
100% Equity, 17.5% Drawdown Rate, Strategy 1	341,444	**492,547**	616,667
100% Equity, 17.5% Drawdown Rate, Strategy 3	341,444	**486,751**	607,306
100% Equity, 15% Drawdown Rate, Strategy 1	327,176	**486,007**	609,637
100% Equity, 15% Drawdown Rate, Strategy 3	337,035	**480,621**	602,585
75% Equity, 17.5% Drawdown Rate, Strategy 1	366,789	**475,371**	573,691
100% Equity, 12.5% Drawdown Rate, Strategy 1	311,005	**474,752**	598,758
100% Equity, 17.5% Drawdown Rate, Strategy 2	341,239	**474,435**	600,145
75% Equity, 17.5% Drawdown Rate, Strategy 3	366,789	**474,259**	568,843
75% Equity, 15% Drawdown Rate, Strategy 3	363,126	**470,832**	567,310
75% Equity, 17.5% Drawdown Rate, Strategy 2	366,603	**469,293**	566,392
100% Equity, 12.5% Drawdown Rate, Strategy 3	328,576	**468,281**	586,511
Level Guaranteed Life Annuity	467,968	**467,968**	467,968
75% Equity, 15% Drawdown Rate, Strategy 1	353,095	**466,336**	564,120
5% Increasing Guaranteed Annuity	464,230	**464,230**	464,230

These results are not unexpected:

- The with-profits annuity benefits from both an expected return increase from having equity exposure as well as the capital usage / mortality credits benefit of a guaranteed annuity.
- The living annuity options that provide the most income are those with the highest drawdown rate and equity exposure. This makes sense – the retiree is maximising his expected return (we are now dealing with means here and not medians) and extracting maximum income from the annuity. As justification of this, the average capital remaining after 30 years for the top living annuity strategy was below R35,000 (compared to the R500,000 starting capital).
- The guaranteed annuities rank above the 50% equity living annuities (irrespective of drawdown rate) and the 100% equity living annuities with 7.5% and 10% drawdown

[13] Further research is probably required into which discount rates to use (for example, given that we are using a real-world asset model should stochastic deflators be used?), but the authors felt the starting yield curve would provide consistency across the annuity types.

rates (which provide similar initial income to the respective guaranteed annuities). This illustrates the mortality credits benefit.
- We should again reinforce that 100% equity portfolios would not actually be available in a living annuity.

One might question why the guaranteed annuities do not have an EPV of income of R500,000 (the initial investment amount). They would have if it were not for the following reasons:
- The projection term used is only 30 years
- They were priced assuming a 10-year guaranteed term

This also explains why the 5% increasing annuity has a slightly lower EPV than the level guaranteed annuity (it has an increasing income stream, and would have benefited more from a longer projection term).

Referring again to the ruin probabilities in Appendix D, we come to a conclusion about living annuities: The living annuity options that provide maximum income also provide little or no chance of maintaining or growing the income over time.
If the retiree wants to maximise his income he needs to choose as high a drawdown rate and equity allocation as possible, but unfortunately this gives a cashflow profile that will almost certainly decrease over time. This is unlikely to meet his needs.

One can look at the future income from the options that were shown to be optimal in terms of ruin probabilities.
- In terms of maintaining a level income:

	20%	**EPV**	**80%**
25% Equity, 7.5% Drawdown Rate, Strategy 2	335,925	**341,247**	346,474

The income bands are relatively narrow, characteristic of this specialised drawdown strategy with a low enough drawdown rate. The EPV of income is well below that of the level guaranteed life annuity (which effectively compares to a 10% drawdown rate).

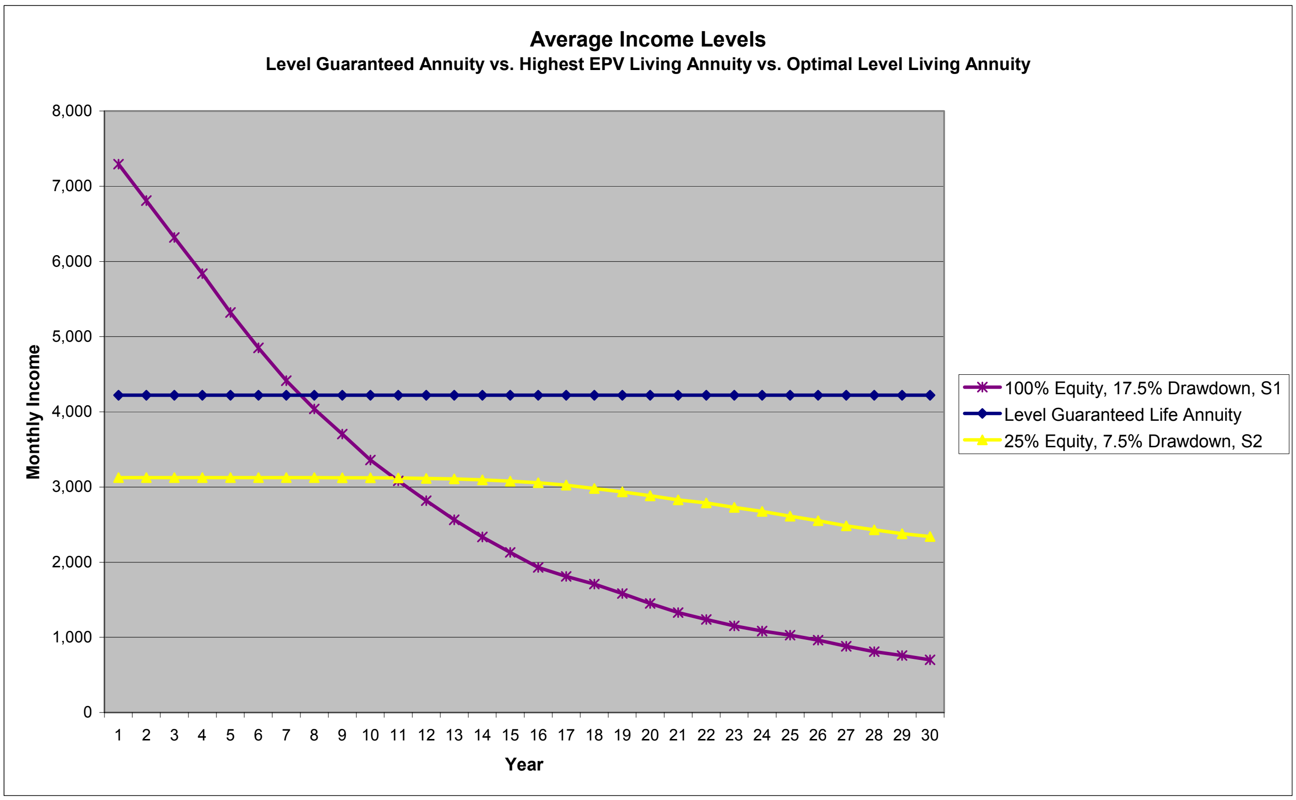
- In terms of growing the income over time (where there were a number of strategies we couldn't distinguish between):

	20%	**EPV**	**80%**
100% Bond, 2.5% Drawdown Rate, Strategy 3	177,861	**183,278**	187,507
25% Equity, 2.5% Drawdown Rate, Strategy 3	178,473	**192,184**	201,942
50% Equity, 2.5% Drawdown Rate, Strategy 3	177,531	**209,340**	233,025
75% Equity, 2.5% Drawdown Rate, Strategy 3	176,361	**230,260**	269,588
100% Equity, 2.5% Drawdown Rate, Strategy 3	174,143	**253,344**	302,385
100% Bond, 5% Drawdown Rate, Strategy 3	315,206	**322,213**	330,040
25% Equity, 5% Drawdown Rate, Strategy 3	314,138	**329,597**	348,484
50% Equity, 5% Drawdown Rate, Strategy 3	299,742	**327,448**	351,846
75% Equity, 5% Drawdown Rate, Strategy 3	280,442	**328,643**	352,339
100% Equity, 5% Drawdown Rate, Strategy 3	256,200	**335,961**	362,204

The income bands are relatively narrow for the more conservative allocations, and widen as the equity exposure and EPV of income increases. The EPV of income increases significantly if the drawdown rate is increased to 5% (with the 20th income percentile still being above the 2.5% drawdown rate's EPV), suggesting the retiree should rather opt for one of the 5% income drawdown options.

The with-profits and 5% increasing guaranteed annuity are still, however, far superior.

Graphing some of the expected income streams for some of the annuity types:

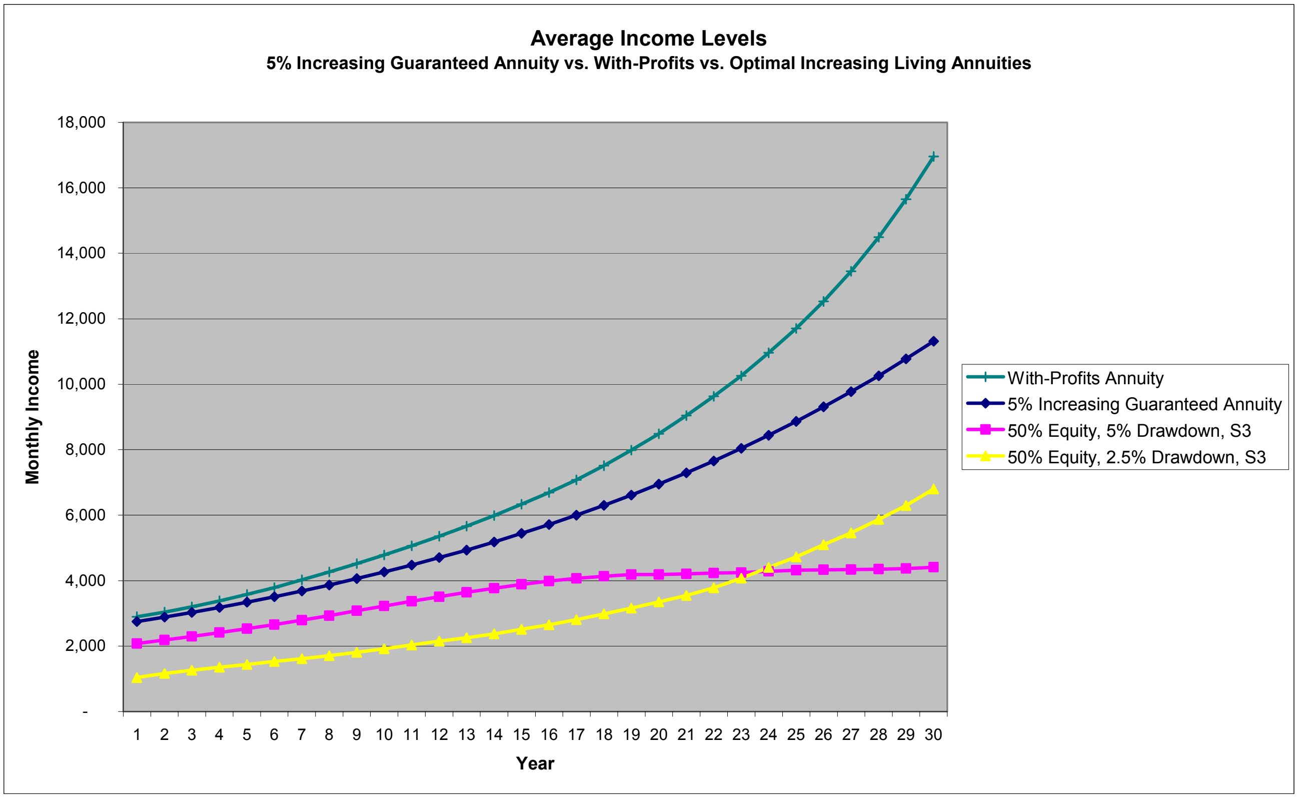


Here one can see the level guaranteed annuity compared to the living annuity that provides maximum total income (100% equity 17.5% drawdown), as well as the living annuity best suited to providing a level income (25% equity, 7.5% drawdown rate, strategy 2).[14]

One can clearly see that in order to maintain one's income in the living annuity, one has to reduce the drawdown rate and sacrifice total income. Expressing this more intuitively, to maintain one's income one needs to limit the income drawdown so that the capital does not decline.

[14] It may look strange that the seemingly optimal living annuity has an income stream that tails off over time. This strategy, however, has a median income that remains level until year 29 (i.e. is actually above the mean income late in the term).

Turning to some of the options conducive to producing an increasing income stream:



This illustrates how the living annuity can support a 5% increasing income as long as the initial drawdown rate is 2.5%. Increasing the initial drawdown to 5% means the average income will eventually tail off (although as mentioned, it still provides greater total lifetime income than the 2.5% initial drawdown option).

We can also see how the with-profits annuity is expected to outperform the 5% increasing guaranteed annuity – the expected present value of income is 14% higher.
There is, however, risk around this.

- We saw from the ruin probabilities that it has just over a 50% chance of not being able to provide a 5% increasing income
- There is a 20% chance that the with-profits annuity provides a present value of income of R386,736, 17% lower than the 5% increasing annuity.

It seems clear that in providing an increasing income stream the with-profits and 5% increasing guaranteed annuity are superior to the living annuity. Which one of the two to choose is less clear, and will eventually depend on the risk-preference of the retiree.

4.3 Switching

A strategy where the retiree invests in a living annuity to begin with, and then switches across to the guaranteed life annuity at a predetermined future age was also tested.
At the switch age (70, 75. 80 and 85 were tested) the remaining living annuity capital is used to purchase a guaranteed life annuity, at that particular return scenarios prevailing annuity rate. [15]

Not all the living annuity asset allocation and drawdown rate options were examined.
- It was assumed the client would draw a fixed % of capital throughout (i.e. drawdown strategy 1), and at a rate of 2.5%, 5%, 7.5% or 10%.
- The 25%, 50% and 75% equity allocation portfolios were used.

Appendix F shows the expected present value, 20th and 80th percentiles of future income, as well as the ruin probabilities (relative to a level income) for these switching scenarios.

Looking at the results for a 50% equity fund and 7.5% drawdown rate, for example, we see the following:

Age at Switch	**EPV of Income**	**Ruin Probability at End of Year**					
		5	**10**	**15**	**20**	**25**	**30**
Never switch	R367,603	45%	51%	57%	60%	57%	55%
70	R415,152	45%	21%	21%	21%	21%	21%
75	R396,923	45%	51%	26%	26%	26%	26%
80	R383,048	45%	51%	57%	31%	31%	31%
85	R374,280	45%	51%	57%	60%	30%	30%

From the above table, it is evident that:
- **Switching out of the living annuity adds value to the retiree (the EPV of income increases, and ruin probability decreases)**
- **In order to derive the most benefit, switching should occur by age 70**

However, even with the switch the EPV of income is still below that of the guaranteed life annuity, which also has zero ruin probability.

So why is this the case, given that some of the overseas research suggested switching at an older age could be optimal?

Firstly, let's look at the extra return achieved in the living annuity. If the equity risk premium were equal to 4.5% and bonds earned 1% more than risk-free, a 50% equity portfolio would be expected to earn a risk premium of about 2.75%. In section 3.5.4 we saw that such a portfolio attracted a total fee of about 2.10%. Thus the premium above risk-free earned by such a fund would be 0.65%, or 65 basis points.

Turning to the mortality credits table in section 2.2.6, we see that mortality credits already dominate the risk premium from age 65 as they are more than the 65 basis points

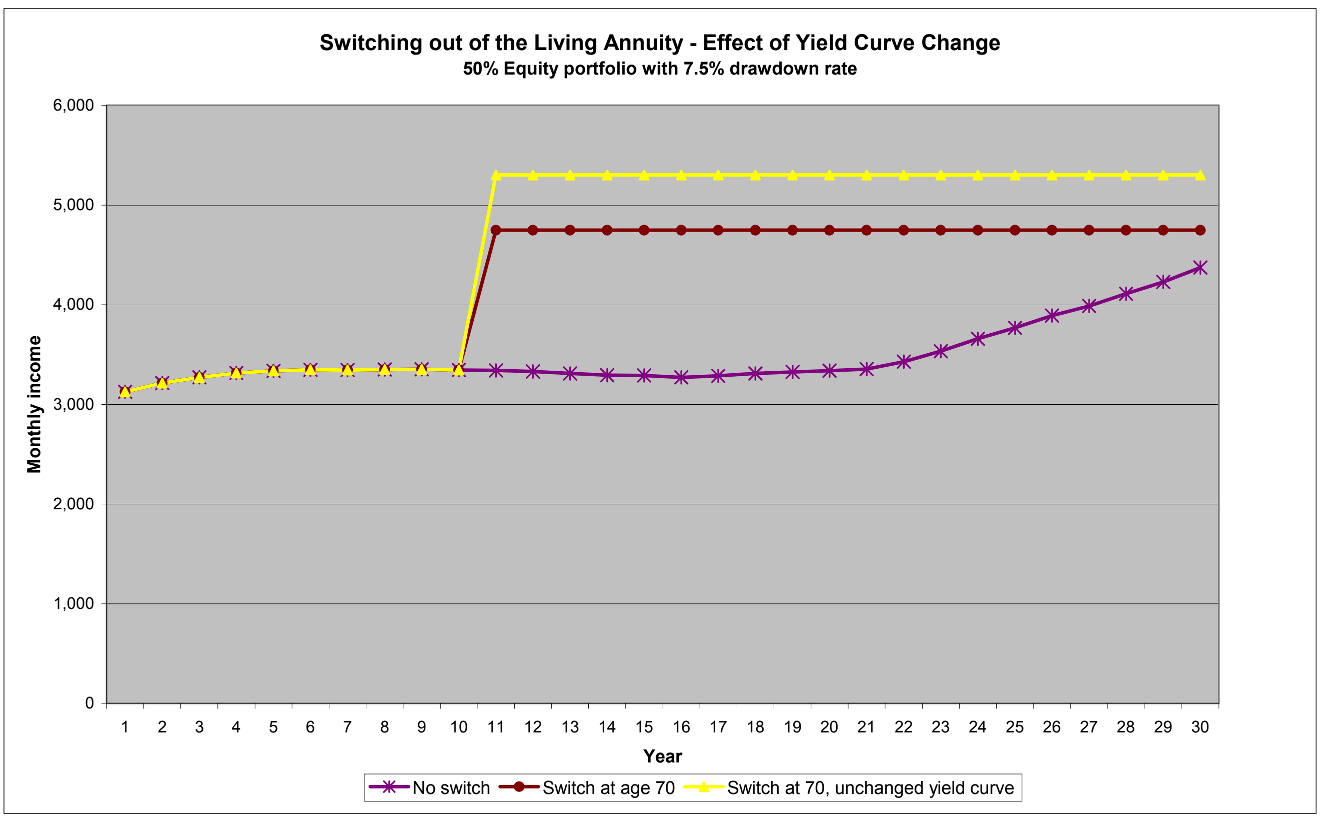
[15] The bases for projecting living annuity capital and calculating future annuity rates are detailed in section 3

offered by the living annuity. This suggests annuitisation before age 65. The stochastic results suggest annuitisation by age 70. Switching before age 70 was not tested – it could well have yielded higher EPVs.

The reason why this is younger than that suggested by overseas researchers is likely to be the extent of the living annuity charges. For example, an advice free[16] index-tracking portfolio might only carry a total charge of 0.75%, meaning a net living annuity risk-premium of 2% (instead of 0.65%). The optimal switching age would then be around 75 for males and closer to 80 for females.

Secondly, we need to bear in mind that the 31 December 2007 yield curve is downward sloping. As discussed in section 2.2.4, this means that, all other things being equal and ignoring the mortality credit effect, future annuity rates are expected to be lower at the date of switch than at retirement.

The graph below illustrates this – it shows the average monthly income from the living annuity, how this would increase on purchase of the guaranteed annuity at age 70, and how the income would have increased even more if the yield curve used in calculating the annuity rate at age 70 were the same as at 31 December 2007.



Note that even in the no switching case the average monthly income across all scenarios increases over time. It may thus seem counter-intuitive that the ruin probability is above

[16] Having said this, it isn't clear how a retiree would be able to implement this strategy without advice.

50%, but the reason for this is that the distribution of income is not symmetrical (the mean is above the median, with the 50^{th} percentile income below the starting income).

The above results suggest that as at 31 December 2007, a 60-year-old male retiree would not have benefited from delaying annuitisation. He should thus have locked into a guaranteed annuity at retirement.

5 Conclusions and Potential Future Research

Retirees are faced with the difficult choice of what to do with their retirement savings. Apart from the fact that no single rule applies to all individuals, there is no clear research in South Africa on which annuity strategies are expected to provide the most income over one's future lifetime. This paper aims to be a first attempt at placing some research on this in the public domain.

This analysis is by no means perfect, and the methodology and assumptions can almost certainly be improved on. For example, the authors feel it is worthwhile examining whether the conclusions hold if:

- Different retirement ages are considered
- Expected future interest rates differ (for example, if there were a flat or upward sloping yield curve).
- The equity risk-premium assumption differs.
- More modern living annuity investment portfolios (e.g. absolute return funds, with limited downside volatility) are chosen.
- A living annuity drawdown option where one only receives the portfolio's income (and never disinvests capital) is chosen.
- More modern with-profit annuity investment strategies (e.g. dynamic hedging) are modelled.
- Inflation risk is specifically modelled (i.e. measuring the likelihood of success of the guaranteed, with-profits, and living annuities against an inflation benchmark).
- Options to leave capital / provide income to dependents are considered

Potentially the most obvious conclusions are about the living annuity.
In order to obtain the maximum expected income, the living annuity investor should opt for the highest available drawdown rate and most aggressive investment strategy. This, however, has two drawbacks:

1. There is a high probability of the future income being significantly lower than expected.
2. The future income stream is virtually certain to be a rapidly decreasing one.

This last point is the most important one – a decreasing cashflow pattern is unlikely to meet a retiree's needs.

To prevent this (and try and obtain a level or growing income stream), the living annuity investor needs to reduce his initial drawdown rate. We have shown that customising or varying the drawdown percentage over time can help in increasing the chance of obtaining the desired income pattern. The research suggests that to have a high probability of maintaining the Rand amount of his income, the retiree should not have an initial drawdown rate of more than 7.5% of his living annuity capital. This should reduce to 2.5% or 5% (depending on risk-preference) if the retiree wants a growing income.

In terms of asset allocation, it seems that a more conservative investment portfolio (25% in equities) is optimal. An interesting finding was the path-dependence of income aspect of the more risky portfolios – disinvesting income during periods of negative returns reduces the ability to benefit from any subsequent positive returns.

The guaranteed (both level and 5% increasing) and with-profits annuity seem to be far superior to the living annuity from a risk-adjusted return point of view - they provide similar / greater expected future lifetime income at far lower risk.
Comparable risk living annuities (for example, the 7.5% drawdown 25% equity allocation living annuity compared to the level guaranteed life annuity) provide far lower expected lifetime income.

We saw that switching from the living to the guaranteed annuity during retirement can add value (both in terms of increasing income and reducing risk). However, because of the relatively low net (after charges) living annuity returns and the downward sloping initial yield curve, it was still optimal to have purchased the guaranteed annuity at retirement.

What is not obvious is whether one should choose the with-profits or one of the guaranteed life annuities. This will eventually have to come down to individual risk-preference – the with-profits annuity is expected to provide around 14% greater lifetime income (in present value terms) than a guaranteed annuity, but has a 20% chance of providing 17% less income (again in present value terms). Alternatively, it has a little more than 50% chance of not being able to grow the income by 5% p.a.

A counter to this is that in high inflation scenarios, the with-profits annuity should be able to offer a great degree of inflation protection (via higher bonus declarations). The guaranteed annuities (especially the level one) would then provide income streams whose purchasing power decreases over time
This is an important consideration in South Africa, and if this were used as the risk benchmark / metric, one might actually find the with-profits annuity comes out lower risk (i.e. with lower ruin probabilities) than the guaranteed annuities. One could even go so far as to say that the level guaranteed annuity should not even be an option – it is certain to provide a decreasing real income stream to the retiree.

The above analysis, however, does not answer the question of what a retiree should do if their initial income needs are above those that can be met by the guaranteed or with-profits annuity. We have seen that choosing a living annuity and selecting a high drawdown rate is not a sustainable solution. No annuity strategy is able to solve this problem, and unfortunately the retiree either needs more retirement capital or needs to decrease their living expenses.

"I advise you to go on living solely to enrage those who are paying your annuities. It is the only pleasure I have left." — Voltaire

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Appendix A: Mortality Basis and Mortality Credits

SAIFL98 and SAIML 98 Standard Tables

Age	Female qx	Male qx
55	0.00442	0.00979
56	0.00483	0.0107
57	0.00528	0.0117
58	0.00578	0.0128
59	0.00633	0.01401
60	0.00694	0.01536
61	0.00761	0.01684
62	0.00835	0.01841
63	0.00917	0.02002
64	0.01007	0.02168
65	0.01105	0.0234
66	0.01213	0.02518
67	0.0133	0.02704
68	0.01458	0.02899
69	0.01594	0.03104
70	0.0174	0.03319
71	0.01905	0.03545
72	0.02092	0.03791
73	0.02304	0.04066
74	0.02544	0.04374
75	0.02815	0.04719
76	0.03122	0.05104
77	0.03469	0.05534
78	0.03862	0.06014
79	0.04305	0.06551
80	0.04806	0.07149
81	0.05371	0.07816
82	0.06009	0.08558
83	0.06727	0.09384
84	0.07535	0.10303
85	0.08442	0.11319
86	0.09431	0.12416
87	0.10507	0.13596
88	0.11671	0.14862
89	0.12927	0.16218
90	0.14276	0.17666
91	0.15718	0.19208
92	0.17255	0.20845
93	0.18884	0.22578
94	0.20605	0.24408
95	0.22414	0.26334
96	0.24306	0.28354
97	0.26278	0.30466
98	0.28321	0.32665
99	0.3043	0.34948
100	0.32595	0.37309
101	0.34807	0.3974
102	0.37056	0.42235
103	0.39332	0.44784
104	0.41623	0.47377
105	0.43918	0.50004
106	0.46206	0.52652
107	0.48476	0.5531
108	0.50716	0.57964
109	0.52916	0.60603
110	0.55067	0.63212

Calculating Mortality Credits

According to the annuity basis used in this paper, a 60-year-old male would have been quoted a monthly annuity of R4,221 for a R500,000 consideration as at 31 December 2007. This annuity includes a ten year certain or guarantee period.

Now suppose the 60 year-old decided to invest the R500,000 in a living annuity arrangement and then withdraw R4,221 per month for the next ten years. What would be the required investment return needed to successfully withdraw R4,221 per month and still have enough capital at the end of ten years to purchase an identical annuity i.e. to purchase an annuity of R4,221 per month?

To compute this return we firstly need the annuity rate for a 70 year-old. Assuming that the yield curve remains unchanged (i.e. isolating the mortality effect), a consideration of R500,000 would purchase a much higher monthly annuity of about R5,590. If the annuity is to be equal to R4221, the required consideration would be 4221/5590×500,000 = R377,549, or roughly 76% of the original cost. The same annuity would be cheaper if purchased later. A 60 year-old male requires R500,000 to generate R4,221 for life (with a 10 year guarantee) while at age 70 the same life would only require R377,549.

Now, the return the living annuity needs to earn to provide the 4,221p.m. and have R377,549 in capital remaining after 10years is found by solving the equation:

$$500{,}000 = 4{,}221 \times 12 \times a_{\overline{10|}}^{(12)} + 377{,}549v^{10} \qquad @\ i$$

The interest rate here works out to 8.92%. As long as the retiree can earn this in the living annuity he can delay annuitisation for another 10 years.
Milevsky (2003) calls i the implied life credit.

Expressing this differently, the 10-year risk-free at 31 December 2007 was 8,35%.
The retiree thus needs to be able to earn a net living annuity return of 0.57%, or 57 basis points, above risk-free.

We define: **Mortality credit =** $i - 8.35\%$

Appendix B: Guaranteed Life Annuity Basis

Market annuity rates
Male
Monthly in arrear — **10 year guaranteed term**
R100,000 consideration — **3 Jan & 27 June = Personal Finance Magazine**
1.71% initial commission — **4 July = Personal Finance Newspaper**
No deduction for tax

	3-Jan	3-Jan	3-Jan	27-Jun	4-Jul	27-Jun	27-Jun	Average	Used
Company / **Age**	60	65	70	60	60	65	70		
Old Mutual	852.61	915.13	986.94	977.36	990.49	1,034.63	1,100.54		
Liberty	857.47	910.11	973.51	940.92	983.54	1,027.93	1,086.23		
Metropolitan	821.96	895.00	974.69	948.58	954.80	1,012.68	1,083.46		
Momentum	805.79	862.41	926.24		975.52*				
Sanlam	815.78	877.47	955.89	935.06	972.13	1,027.84	1,098.99		
Average	**830.72**	**892.02**	**963.45**	**950.48**	**975.30**	**1,025.77**	**1,092.31**		
RIY vs. Swap	**1.25%**	**1.31%**	**1.35%**	**1.40%**	**1.25%**	**1.12%**	**1.11%**	**1.26%**	
RIY vs. Bond	**0.68%**	**0.64%**	**0.58%**	**0.89%**	**0.58%**	**0.54%**	**0.42%**	**0.62%****	**0.60%**
MWR (with commission)	**93.5%**	**94.2%**	**95.0%**	**92.8%**	**94.7%**	**95.3%**	**96.2%**	**94.5%**	

**Incorrect rate of R875.52 reported.*
*** Average of 0.57% if the 0.89% 27 June outlier is removed*

Appendix C: Living Annuity Fee References

General Equity Funds	Base Asset Management Fee*
Allan Gray Equity Fund	1.71%
Coronation Equity Fund	1.43%
Nedgroup Rainmaker Fund	1.71%
Old Mutual Investors Fund	1.54%
Sanlam General Equity Fund	1.42%
Average	**1.56%**

** Open retail class funds, assuming the funds achieve benchmark returns (i.e. excluding performance fees).*

Bond Funds	Asset Management Fee
Coronation Bond	0.86%
Old Mutual Gilt	0.86%
Stanlib Bond	0.86%
	0.86%

Source: Company websites

Appendix D: Ruin Probabilities

1. Likelihood of Not Maintaining the Initial Rand Amount of Income

Drawdown Strategy 1 (same drawdown %)

100% Bond	**End of Year**						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	9%	2%	0%	0%	0%	0%
5.0%	0%	22%	15%	17%	20%	14%	11%
7.5%	0%	49%	**69%**	**87%**	**90%**	**86%**	**85%**
10.0%	0%	**78%**	**99%**	**100%**	**99%**	**98%**	**97%**
12.5%	0%	**94%**	**100%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

25% Equity	**End of Year**						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	6%	3%	1%	1%	0%	0%
5.0%	0%	20%	16%	16%	17%	14%	11%
7.5%	0%	45%	**57%**	**66%**	**72%**	**68%**	**67%**
10.0%	0%	**71%**	**90%**	**95%**	**96%**	**95%**	**94%**
12.5%	0%	**90%**	**99%**	**100%**	**99%**	**99%**	**99%**
15.0%	0%	**98%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

50% Equity	**End of Year**						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	14%	9%	7%	6%	3%	2%
5.0%	0%	27%	26%	26%	25%	21%	19%
7.5%	0%	45%	**51%**	**57%**	**60%**	**57%**	**55%**
10.0%	0%	**64%**	**76%**	**84%**	**87%**	**86%**	**84%**
12.5%	0%	**80%**	**92%**	**96%**	**96%**	**97%**	**97%**
15.0%	0%	**90%**	**98%**	**99%**	**99%**	**99%**	**99%**
17.5%	0%	**96%**	**100%**	**100%**	**100%**	**100%**	**100%**

75% Equity	**End of Year**						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	23%	18%	14%	14%	10%	7%
5.0%	0%	34%	32%	33%	32%	28%	26%
7.5%	0%	47%	**51%**	**55%**	**57%**	**55%**	**54%**
10.0%	0%	**60%**	**69%**	**75%**	**78%**	**78%**	**77%**
12.5%	0%	**72%**	**83%**	**90%**	**92%**	**91%**	**92%**
15.0%	0%	**82%**	**93%**	**96%**	**97%**	**98%**	**98%**
17.5%	0%	**89%**	**97%**	**99%**	**99%**	**99%**	**100%**

100% Equity	**End of Year**						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	30%	26%	24%	22%	18%	16%
5.0%	0%	40%	38%	41%	40%	35%	34%
7.5%	0%	50%	**55%**	**56%**	**58%**	**56%**	**56%**
10.0%	0%	**59%**	**67%**	**72%**	**75%**	**76%**	**74%**
12.5%	0%	**69%**	**79%**	**84%**	**87%**	**88%**	**88%**
15.0%	0%	**77%**	**87%**	**93%**	**95%**	**96%**	**96%**
17.5%	0%	**84%**	**93%**	**97%**	**98%**	**99%**	**99%**

Drawdown Strategy 2 (same Rand drawdown)

100% Bond	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	0%	0%	0%	0%	0%	0%
5.0%	0%	0%	0%	0%	0%	0%	0%
7.5%	0%	0%	1%	7%	34%	**79%**	**97%**
10.0%	0%	7%	**52%**	**98%**	**100%**	**100%**	**100%**
12.5%	0%	**51%**	**100%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**95%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

25% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	0%	0%	0%	0%	0%	0%
5.0%	0%	0%	0%	0%	0%	1%	1%
7.5%	0%	0%	1%	11%	26%	42%	**51%**
10.0%	0%	4%	42%	**80%**	**94%**	**97%**	**98%**
12.5%	0%	46%	**93%**	**99%**	**100%**	**100%**	**100%**
15.0%	0%	**91%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

50% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	0%	0%	0%	0%	0%	0%
5.0%	0%	0%	0%	1%	4%	6%	8%
7.5%	0%	0%	9%	22%	35%	44%	47%
10.0%	0%	12%	44%	**66%**	**77%**	**82%**	**83%**
12.5%	0%	47%	**80%**	**91%**	**95%**	**96%**	**97%**
15.0%	0%	**80%**	**96%**	**99%**	**99%**	**99%**	**99%**
17.5%	0%	**95%**	**99%**	**100%**	**100%**	**100%**	**100%**

75% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	0%	0%	0%	1%	1%	1%
5.0%	0%	0%	2%	7%	12%	16%	18%
7.5%	0%	3%	18%	32%	41%	47%	50%
10.0%	0%	21%	46%	**62%**	**70%**	**75%**	**76%**
12.5%	0%	49%	**72%**	**82%**	**86%**	**89%**	**90%**
15.0%	0%	**72%**	**87%**	**92%**	**95%**	**95%**	**96%**
17.5%	0%	**86%**	**95%**	**97%**	**98%**	**98%**	**98%**

100% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	0%	0%	2%	4%	5%	6%
5.0%	0%	1%	8%	16%	21%	26%	28%
7.5%	0%	9%	26%	39%	48%	**52%**	**54%**
10.0%	0%	29%	49%	**62%**	**68%**	**72%**	**74%**
12.5%	0%	**50%**	**68%**	**77%**	**81%**	**84%**	**85%**
15.0%	0%	**68%**	**81%**	**86%**	**90%**	**92%**	**92%**
17.5%	0%	**81%**	**89%**	**93%**	**94%**	**95%**	**95%**

With-Profits Annuity	End of Year						
	Start	**5**	**10**	**15**	**20**	**25**	**30**
	0%	0%	0%	0%	0%	0%	0%

2. *Likelihood of Not Achieving a 5% Increasing Income*

Drawdown Strategy 1 (same drawdown %)

100% Bond	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	46%	**64%**	**84%**	**88%**	**83%**	**81%**
5.0%	0%	**77%**	**99%**	**100%**	**99%**	**98%**	**97%**
7.5%	0%	**93%**	**100%**	**100%**	**100%**	**100%**	**100%**
10.0%	0%	**99%**	**100%**	**100%**	**100%**	**100%**	**100%**
12.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

25% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	44%	**53%**	**63%**	**69%**	**64%**	**61%**
5.0%	0%	**69%**	**88%**	**94%**	**95%**	**93%**	**92%**
7.5%	0%	**89%**	**99%**	**99%**	**99%**	**99%**	**99%**
10.0%	0%	**98%**	**100%**	**100%**	**100%**	**100%**	**100%**
12.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

50% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	43%	49%	**54%**	**57%**	**53%**	**51%**
5.0%	0%	**62%**	**74%**	**82%**	**85%**	**83%**	**82%**
7.5%	0%	**79%**	**91%**	**95%**	**96%**	**96%**	**96%**
10.0%	0%	**89%**	**97%**	**99%**	**99%**	**99%**	**99%**
12.5%	0%	**96%**	**99%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**99%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

75% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	46%	**50%**	**53%**	**55%**	**52%**	**51%**
5.0%	0%	**58%**	**68%**	**73%**	**76%**	**76%**	**75%**
7.5%	0%	**71%**	**82%**	**89%**	**91%**	**89%**	**90%**
10.0%	0%	**81%**	**91%**	**96%**	**96%**	**98%**	**98%**
12.5%	0%	**88%**	**96%**	**98%**	**99%**	**99%**	**99%**
15.0%	0%	**94%**	**99%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**98%**	**99%**	**100%**	**100%**	**100%**	**100%**

100% Equity	End of Year						
Drawdown Rate	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	0%	49%	**53%**	**55%**	**56%**	**54%**	**53%**
5.0%	0%	**58%**	**66%**	**71%**	**73%**	**73%**	**72%**
7.5%	0%	**67%**	**78%**	**83%**	**86%**	**86%**	**86%**
10.0%	0%	**76%**	**86%**	**92%**	**94%**	**94%**	**95%**
12.5%	0%	**83%**	**93%**	**96%**	**97%**	**98%**	**99%**
15.0%	0%	**88%**	**96%**	**99%**	**99%**	**99%**	**100%**
17.5%	0%	**93%**	**98%**	**100%**	**100%**	**100%**	**100%**

Drawdown Strategy 3 (5% increasing Rand drawdown)

100% Bond Drawdown Rate	Start	End of Year 5	10	15	20	25	30
2.5%	0%	0%	0%	0%	0%	0%	0%
5.0%	0%	0%	0%	9%	**84%**	**99%**	**100%**
7.5%	0%	1%	**54%**	**100%**	**100%**	**100%**	**100%**
10.0%	0%	33%	**100%**	**100%**	**100%**	**100%**	**100%**
12.5%	0%	**93%**	**100%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

25% Equity Drawdown Rate	Start	End of Year 5	10	15	20	25	30
2.5%	0%	0%	0%	0%	0%	1%	1%
5.0%	0%	0%	0%	12%	**53%**	**78%**	**88%**
7.5%	0%	1%	44%	**94%**	**99%**	**100%**	**100%**
10.0%	0%	31%	**97%**	**100%**	**100%**	**100%**	**100%**
12.5%	0%	**89%**	**100%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

50% Equity Drawdown Rate	Start	End of Year 5	10	15	20	25	30
2.5%	0%	0%	0%	0%	2%	6%	11%
5.0%	0%	0%	5%	23%	48%	**63%**	**71%**
7.5%	0%	4%	45%	**77%**	**90%**	**94%**	**96%**
10.0%	0%	36%	**85%**	**98%**	**99%**	**99%**	**100%**
12.5%	0%	**78%**	**98%**	**100%**	**100%**	**100%**	**100%**
15.0%	0%	**96%**	**100%**	**100%**	**100%**	**100%**	**100%**
17.5%	0%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

75% Equity Drawdown Rate	Start	End of Year 5	10	15	20	25	30
2.5%	0%	0%	0%	3%	8%	15%	20%
5.0%	0%	0%	13%	33%	50%	**60%**	**67%**
7.5%	0%	12%	48%	**71%**	**81%**	**86%**	**89%**
10.0%	0%	42%	**77%**	**89%**	**95%**	**96%**	**97%**
12.5%	0%	**70%**	**92%**	**98%**	**98%**	**99%**	**99%**
15.0%	0%	**88%**	**97%**	**99%**	**99%**	**99%**	**100%**
17.5%	0%	**96%**	**99%**	**100%**	**100%**	**100%**	**100%**

100% Equity Drawdown Rate	Start	End of Year 5	10	15	20	25	30
2.5%	0%	0%	2%	8%	18%	25%	30%
5.0%	0%	3%	20%	40%	**53%**	**62%**	**67%**
7.5%	0%	20%	**50%**	**69%**	**79%**	**84%**	**85%**
10.0%	0%	46%	**74%**	**85%**	**90%**	**93%**	**94%**
12.5%	0%	**67%**	**87%**	**93%**	**96%**	**96%**	**97%**
15.0%	0%	**82%**	**94%**	**97%**	**98%**	**98%**	**99%**
17.5%	0%	**90%**	**97%**	**99%**	**99%**	**99%**	**99%**

With-Profits Annuity	Start	End of Year 5	10	15	20	25	30
	0%	**50%**	**55%**	**57%**	**61%**	**63%**	**66%**

Appendix E: Present Value of Income

Drawdown Strategy 1 (same drawdown %)

100% Bond			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	157,692	**165,052**	171,897
5.0%	252,908	**265,800**	277,837
7.5%	310,892	**329,222**	346,600
10.0%	348,085	**370,577**	392,355
12.5%	372,179	**398,570**	423,001
15.0%	389,120	**418,236**	445,189
17.5%	402,603	**432,544**	461,670

25% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	156,383	**177,725**	196,186
5.0%	251,712	**283,142**	310,353
7.5%	312,101	**347,453**	380,201
10.0%	350,952	**388,038**	422,928
12.5%	376,044	**414,636**	450,606
15.0%	393,941	**432,765**	468,379
17.5%	406,277	**445,601**	482,471

50% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	146,736	**192,052**	231,122
5.0%	237,997	**302,531**	357,809
7.5%	295,818	**367,603**	432,078
10.0%	333,669	**407,119**	474,019
12.5%	359,468	**432,005**	499,490
15.0%	376,318	**448,316**	514,540
17.5%	389,191	**459,452**	523,878

75% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	135,681	**209,418**	268,212
5.0%	219,679	**325,839**	408,807
7.5%	273,169	**391,608**	485,804
10.0%	309,671	**429,638**	527,943
12.5%	335,565	**452,312**	550,178
15.0%	353,095	**466,336**	564,120
17.5%	366,789	**475,371**	573,691

100% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	121,951	**229,495**	301,907
5.0%	199,409	**352,522**	457,755
7.5%	248,513	**418,787**	536,422
10.0%	285,046	**454,833**	583,275
12.5%	311,005	**474,752**	598,758
15.0%	327,176	**486,007**	609,637
17.5%	341,444	**492,547**	616,667

Drawdown Strategy 2 (same Rand drawdown)

100% Bond			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	158,074	**165,466**	172,208
5.0%	230,983	**232,005**	231,901
7.5%	336,751	**340,831**	345,176
10.0%	374,754	**393,093**	411,005
12.5%	391,033	**416,508**	441,845
15.0%	398,447	**427,538**	455,272
17.5%	402,603	**432,232**	461,194

25% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	156,744	**178,040**	196,224
5.0%	230,983	**237,914**	241,519
7.5%	335,925	**341,247**	346,474
10.0%	378,284	**405,367**	432,855
12.5%	395,117	**430,466**	465,033
15.0%	403,378	**441,086**	476,749
17.5%	406,277	**445,284**	481,739

50% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	148,031	**193,252**	231,122
5.0%	230,983	**250,384**	263,926
7.5%	324,610	**339,227**	346,906
10.0%	360,011	**405,628**	455,602
12.5%	376,415	**438,951**	500,629
15.0%	385,549	**452,947**	514,293
17.5%	389,191	**458,342**	521,374

75% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	139,702	**212,626**	268,212
5.0%	230,983	**266,898**	291,535
7.5%	305,529	**341,928**	354,790
10.0%	337,706	**402,429**	461,966
12.5%	354,536	**439,748**	530,804
15.0%	362,332	**459,799**	553,465
17.5%	366,603	**469,293**	566,392

100% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	129,447	**235,295**	302,276
5.0%	225,112	**285,997**	317,111
7.5%	282,534	**349,368**	367,901
10.0%	311,534	**401,587**	461,966
12.5%	326,522	**437,869**	545,342
15.0%	335,802	**460,977**	579,486
17.5%	341,239	**474,435**	600,145

Drawdown Strategy 3 (5% increasing Rand drawdown)

100% Bond			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	177,861	**183,278**	187,507
5.0%	315,206	**322,213**	330,040
7.5%	361,770	**380,830**	399,523
10.0%	382,019	**408,211**	433,412
12.5%	393,818	**422,186**	448,861
15.0%	400,390	**429,282**	458,246
17.5%	402,603	**432,458**	461,504

25% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	178,473	**192,184**	201,942
5.0%	314,138	**329,597**	348,484
7.5%	364,334	**396,519**	427,709
10.0%	387,520	**423,650**	458,982
12.5%	398,233	**436,578**	472,058
15.0%	403,872	**442,832**	479,395
17.5%	406,277	**445,511**	482,053

50% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	177,531	**209,340**	233,025
5.0%	299,742	**327,448**	351,846
7.5%	346,971	**404,370**	462,038
10.0%	367,370	**436,633**	499,671
12.5%	379,521	**450,446**	512,604
15.0%	386,537	**456,611**	519,608
17.5%	389,191	**459,194**	523,365

75% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	176,361	**230,260**	269,588
5.0%	280,442	**328,643**	352,339
7.5%	322,574	**404,485**	492,137
10.0%	344,834	**443,700**	535,895
12.5%	356,543	**462,268**	552,065
15.0%	363,126	**470,832**	567,310
17.5%	366,789	**474,259**	568,843

100% Equity			
Drawdown Rate	**20%**	**EPV**	**80%**
2.5%	174,143	**253,344**	302,385
5.0%	256,200	**335,961**	362,204
7.5%	296,066	**403,606**	506,124
10.0%	317,468	**444,815**	565,477
12.5%	328,576	**468,281**	586,511
15.0%	337,035	**480,621**	602,585
17.5%	341,444	**486,751**	607,306

Guaranteed Life Annuities

	20%	EPV	80%
Level	467,968	**467,968**	467,968
5% Increasing	464,230	**464,230**	464,230

With-Profits Annuity

With-Profits Annuity	20%	EPV	80%
	386,736	**528,652**	630,483

Appendix F: Switching Results

Switching at Age 70

25% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	301,310	**352,238**	397,780	0%	6%	0%	0%	0%	0%	0%
5.0%	331,053	**376,217**	417,295	0%	20%	0%	0%	0%	0%	0%
7.5%	355,398	**396,124**	434,265	0%	45%	7%	7%	7%	7%	7%
10.0%	374,299	**412,629**	449,420	0%	**71%**	**87%**	**87%**	**87%**	**87%**	**87%**

50% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	273,988	**376,008**	462,151	0%	14%	0%	0%	0%	0%	0%
5.0%	305,980	**397,464**	474,730	0%	27%	1%	1%	1%	1%	1%
7.5%	330,376	**415,152**	488,968	0%	45%	21%	21%	21%	21%	21%
10.0%	349,677	**429,706**	500,561	0%	**64%**	**72%**	**72%**	**72%**	**72%**	**72%**

75% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	242,937	**402,918**	522,574	0%	23%	0%	0%	0%	0%	0%
5.0%	275,637	**421,468**	529,717	0%	34%	4%	4%	4%	4%	4%
7.5%	302,637	**436,606**	541,955	0%	47%	28%	28%	28%	28%	28%
10.0%	322,683	**448,922**	550,944	0%	**60%**	**66%**	**66%**	**66%**	**66%**	**66%**

Switching at Age 75

25% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	248,528	**298,790**	337,306	0%	6%	3%	0%	0%	0%	0%
5.0%	300,444	**342,139**	378,527	0%	20%	16%	0%	0%	0%	0%
7.5%	336,266	**375,117**	410,108	0%	45%	**57%**	16%	16%	16%	16%
10.0%	362,126	**400,263**	436,760	0%	**71%**	**90%**	**89%**	**89%**	**89%**	**89%**

50% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	226,101	**327,112**	408,846	0%	14%	9%	0%	0%	0%	0%
5.0%	277,199	**366,956**	440,800	0%	27%	26%	1%	1%	1%	1%
7.5%	314,628	**396,923**	465,365	0%	45%	**51%**	26%	26%	26%	26%
10.0%	341,335	**419,484**	487,739	0%	**64%**	**76%**	**75%**	**75%**	**75%**	**75%**

75% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	202,221	**360,572**	469,839	0%	23%	18%	0%	0%	0%	0%
5.0%	251,731	**396,033**	501,126	0%	34%	32%	6%	6%	6%	6%
7.5%	289,200	**422,277**	523,988	0%	47%	**51%**	34%	34%	34%	34%
10.0%	317,334	**441,677**	540,078	0%	**60%**	**69%**	**68%**	**68%**	**68%**	**68%**

Switching at Age 80

25% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	206,485	**249,890**	279,590	0%	6%	3%	1%	0%	0%	0%
5.0%	275,511	**314,966**	346,375	0%	20%	16%	16%	0%	0%	0%
7.5%	324,007	**360,632**	393,561	0%	45%	**57%**	**66%**	25%	25%	25%
10.0%	355,514	**392,975**	428,258	0%	**71%**	**90%**	**95%**	**88%**	**88%**	**88%**

50% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	189,310	**278,191**	337,469	0%	14%	9%	7%	0%	0%	0%
5.0%	258,323	**340,242**	406,641	0%	27%	26%	26%	2%	2%	2%
7.5%	303,879	**383,048**	448,768	0%	45%	**51%**	**57%**	31%	31%	31%
10.0%	336,993	**412,792**	479,749	0%	**64%**	**76%**	**84%**	**76%**	**76%**	**76%**

75% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	169,722	**313,533**	396,052	0%	23%	18%	14%	0%	0%	0%
5.0%	236,432	**371,176**	467,210	0%	34%	32%	33%	7%	7%	7%
7.5%	279,625	**410,023**	509,169	0%	47%	**51%**	**55%**	37%	37%	37%
10.0%	311,757	**436,299**	534,684	0%	**60%**	**69%**	**75%**	**70%**	**70%**	**70%**

Switching at Age 85

25% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	178,575	**211,783**	233,318	0%	6%	3%	1%	1%	0%	0%
5.0%	263,152	**296,935**	325,497	0%	20%	16%	16%	17%	0%	0%
7.5%	316,703	**352,603**	385,286	0%	45%	**57%**	**66%**	**72%**	24%	24%
10.0%	352,582	**389,716**	424,193	0%	**71%**	**90%**	**95%**	**96%**	**87%**	**87%**

50% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	166,214	**235,714**	280,680	0%	14%	9%	7%	6%	0%	0%
5.0%	248,131	**320,296**	381,617	0%	27%	26%	26%	25%	1%	1%
7.5%	299,430	**374,280**	437,791	0%	45%	**51%**	**57%**	**60%**	30%	30%
10.0%	335,308	**409,319**	476,444	0%	**64%**	**76%**	**84%**	**87%**	**76%**	**76%**

75% Equity	PV of Income			Ruin Probability						
Drawdown Rate	**20%**	**EPV**	**80%**	**Start**	**5**	**10**	**15**	**20**	**25**	**30**
2.5%	151,312	**265,770**	333,448	0%	23%	18%	14%	14%	0%	0%
5.0%	228,355	**348,922**	439,368	0%	34%	32%	33%	32%	5%	5%
7.5%	276,747	**400,370**	494,005	0%	47%	**51%**	**55%**	**57%**	34%	34%
10.0%	310,810	**432,576**	531,300	0%	**60%**	**69%**	**75%**	**78%**	**71%**	**71%**

*** All ruin probabilities are in respect of not being able to maintain the initial Rand amount of income*