Modelling the Market in a Risk-averse World

(work in progress)

R.J. Thomson

Actuarial Society Convention 2008

Modelling the Market in a Risk-averse World

- Introduction
- Models of the market portfolio
- The parameterisation of the models
- Summary

Introduction

- SA data: 1987–2007
- ‘returns’: real annual forces of return
- ‘market portfolio’: listed equity & government bonds
- Conditionally on information at the start of the year:
  - return on market portfolio normally distributed;
  - market price of risk reasonably greater than 0

Introduction

- Purpose of descriptive models: to inform the definition of predictive models
- Purpose of estimation: to derive ex-post estimates of ex-ante parameters
- Rational-expectations hypothesis: applied so far as possible
- Risk-free rate not primarily an explanatory variable: primarily to establish a minimum ex-ante expected value of return on market portfolio
- Not an attempt to obtain ‘the real-world’ model

Models of the market portfolio: the basic model

$$\delta_{M;t} = g\delta_{I;t} + h + \sigma\varepsilon_t;$$

where:

$\delta_{I;t}$ is the risk-free rate;

$g \geq g^* \geq 1$ and $h \geq h^* \geq 0$;

$g^* = 1.2$ and $h^* = 0$; or $g^* = 1$ and $h^* = 0,01$

$\varepsilon_t \overset{i.i.d.}{\sim} N(0,1).$

Models of the market portfolio: the regime-switching model

$$\delta_{M;t} = g_{S_t}\delta_{I;t} + h_{S_t} + \sigma_{S_t}\varepsilon_t$$

where:

$S_t \in \{0,1\}$;

$\Pr\{S_t = 0 \mid S_{t-1} = 0\} = p_{00}$;

$\Pr\{S_t = 1 \mid S_{t-1} = 0\} = p_{01} = 1 - p_{00}$;

$\Pr\{S_t = 0 \mid S_{t-1} = 1\} = p_{10}$; and

$\Pr\{S_t = 1 \mid S_{t-1} = 1\} = p_{11} = 1 - p_{10}$.

$g_s \geq g_s^* \geq 1$ and $h_s \geq h_s^* \geq 0$;

$g_s^* = 1,2$ and $h_s^* = 0$; or $g^* = 1$ and $h^* = 0,01$

Models of the market portfolio: the exponential autoregressive model

$$\delta_{M;t} = g\delta_{I;t} + h\exp\left\{\alpha\left(\delta_{M;t-1} - g\delta_{I;t-1}\right)\right\} + \sigma_M \varepsilon_t$$

Models of the market portfolio: the ARCH model

$$\delta_{M;t} = g\delta_{I;t} + h + z_t$$

where:

$$z_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = a + b z_{t-1}^2$$

The parameterisation of the models

- Maximum-likelihood estimates
- 95% confidence limits
- Akaike Information Criterion:

$$A = 2k - 2l$$

- Mean market price of risk:

$$\overline{R} = \frac{\hat{\mu}_M - \overline{\delta}_I}{\hat{\sigma}_M}$$

- Bias:

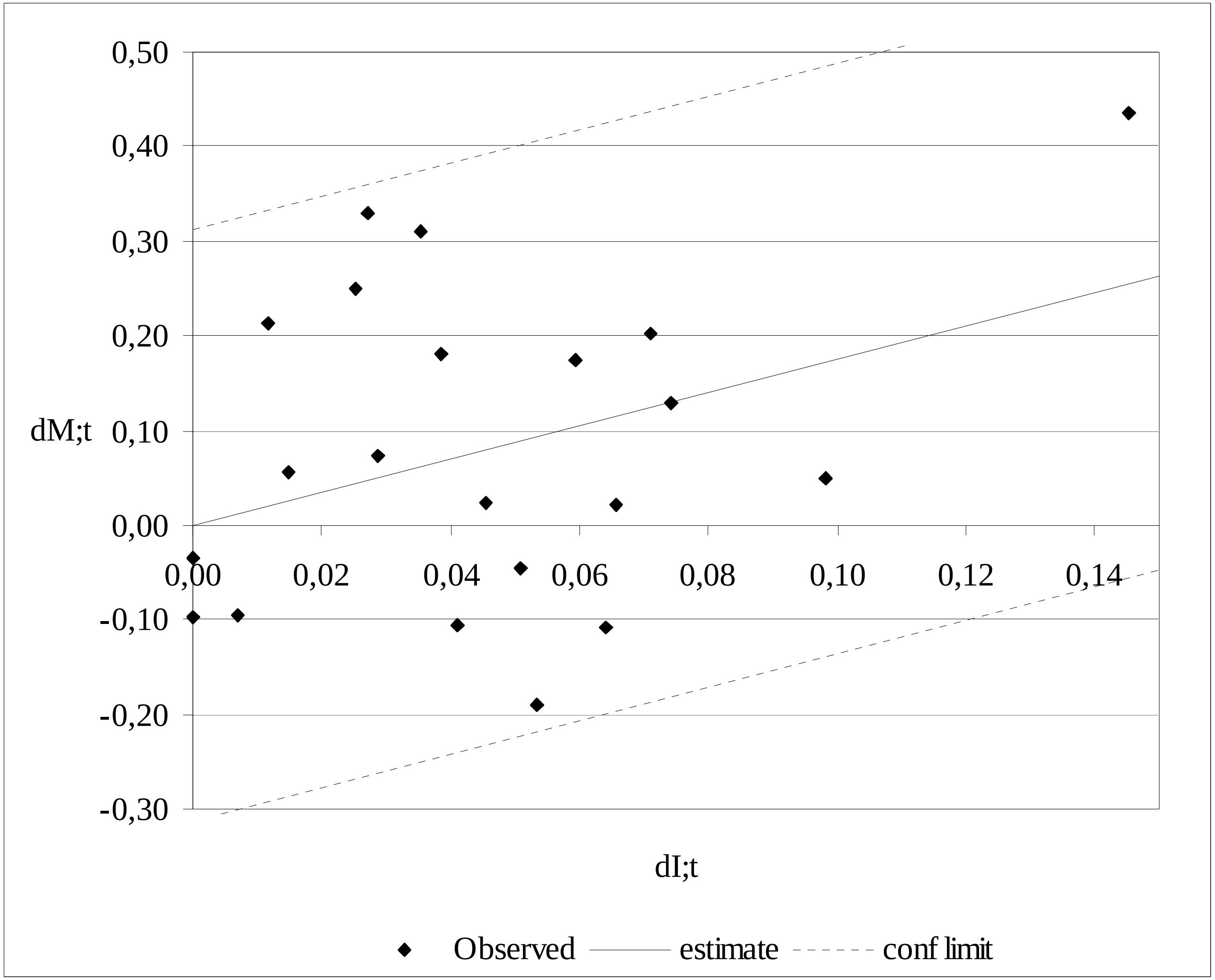
$$B = \overline{\delta}_M - \hat{\mu}_M$$

Parameterisation: basic model

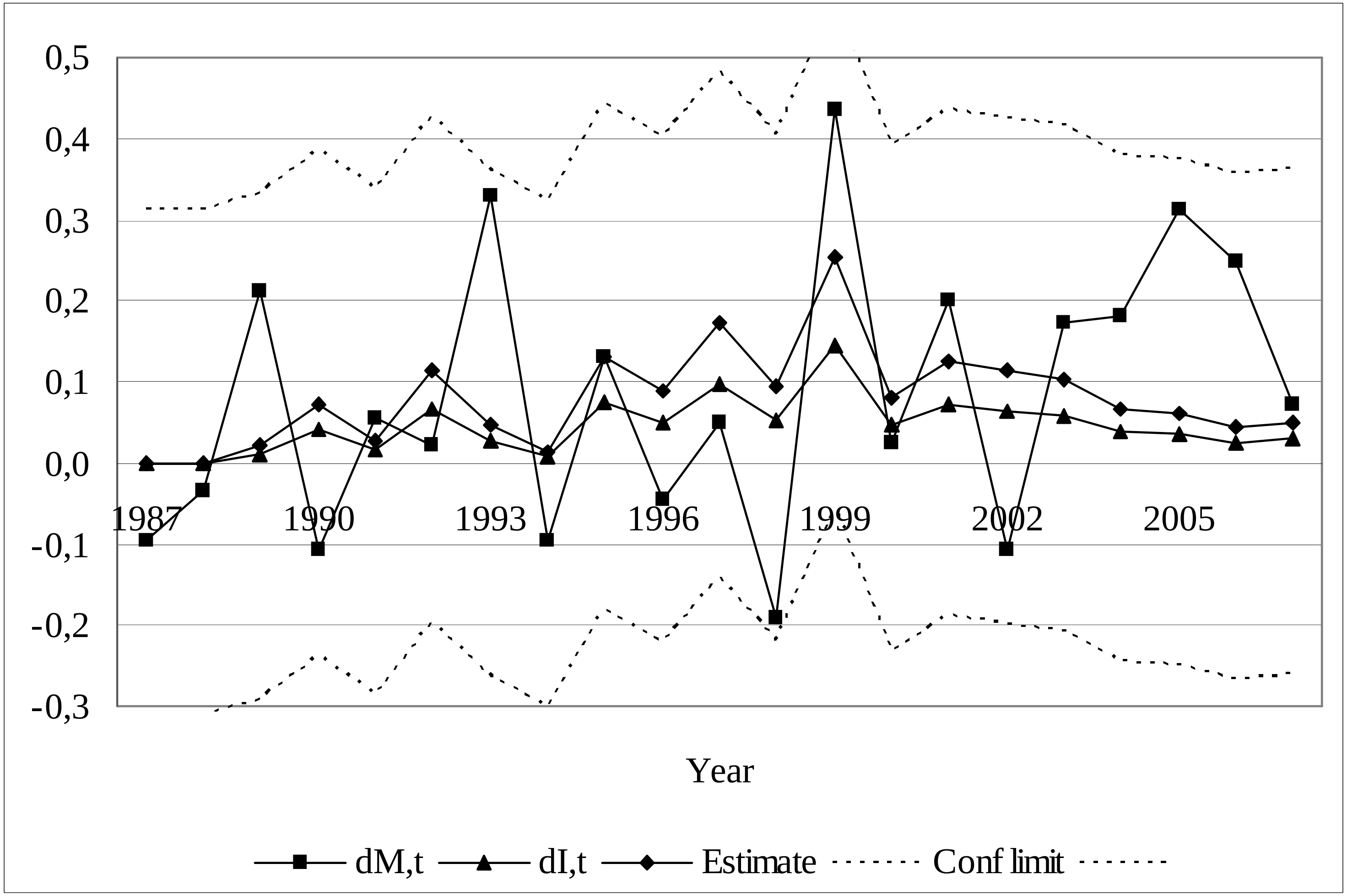
Parameter	Details	constraints		
		basic	$g = 1$	$h = 0$
g	estimate	1,59		1,76
	confidence limits	1; 3,8		1; 2,9
h	estimate	0,012	0,039	
	confidence limits	0; 0,14	0; 0,11	
σ_M	estimate	0,163	0,160	0,159
	confidence limits	0,11; 0,21	0,11; 0,21	0,11; 0,20
k		3	2	2
l		9,28	9,15	9,30
A		–12,56	–14,30	–14,60
R		0,24	0,24	0,22
B		0	0	0,004

Parameterisation: basic model:

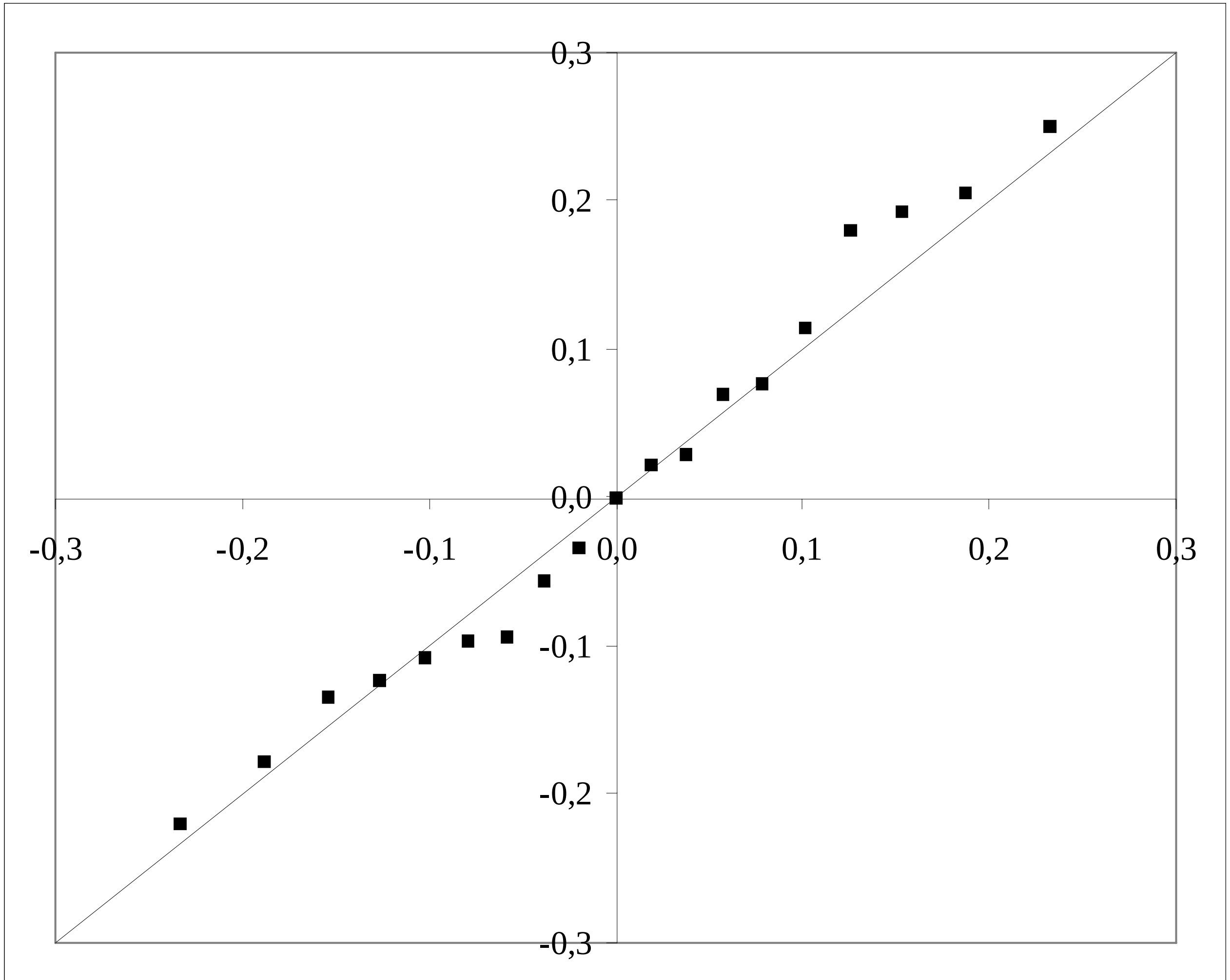
$\delta_{M:t}$ *vs.* $\delta_{I:t}$



Parameterisation: basic model: time series



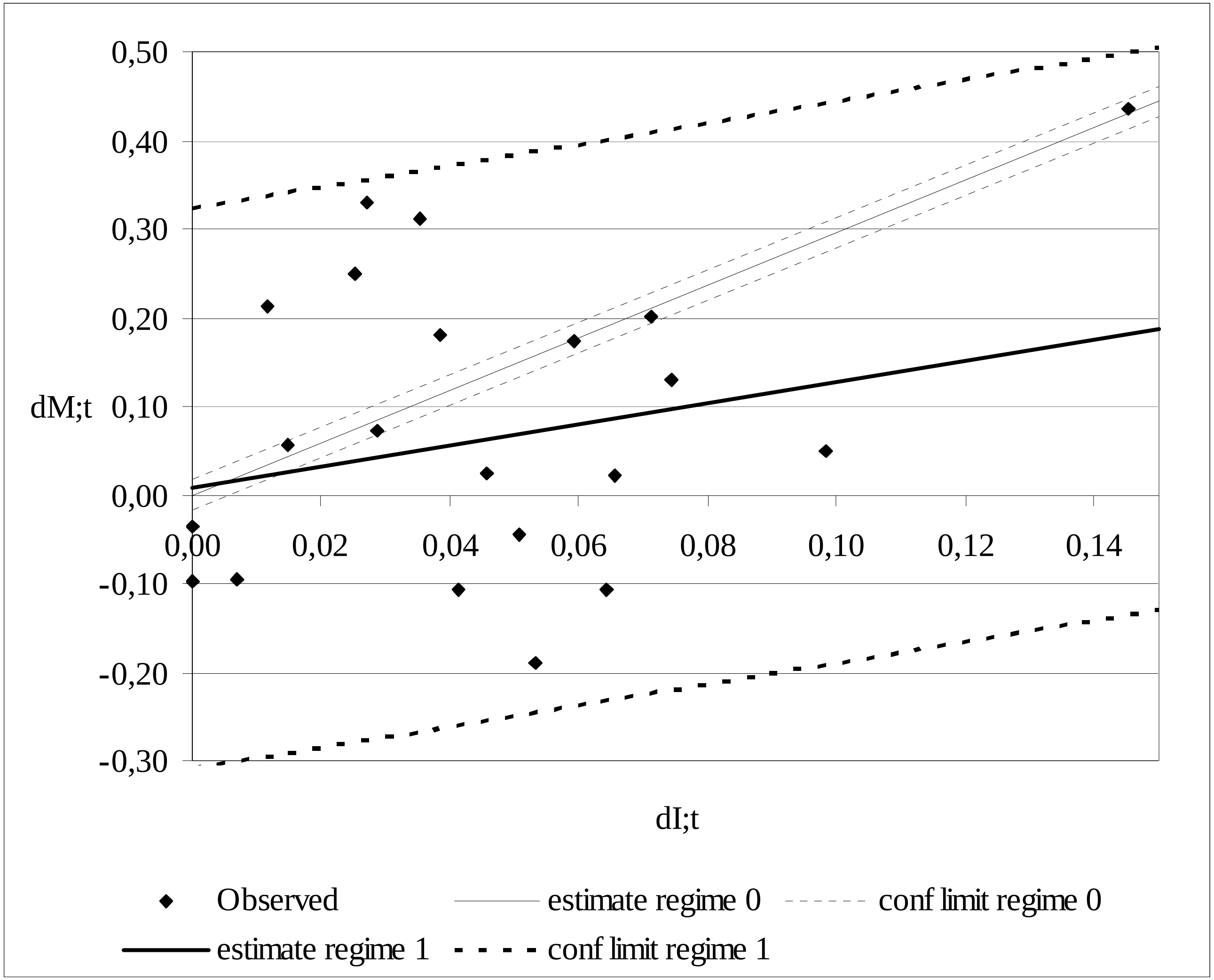
Parameterisation: basic model: Q-Q plot



Parameterisation: regime-switching model

Parameter	
p_{00}	0
p_{10}	0,24
g_0	2,96
h_0	0
σ_0	0,009
g_1	1,2
h_1	0,007
σ_1	0,162
l	13,25
k	7
A	–12,50
R	0,56
B	–0,051

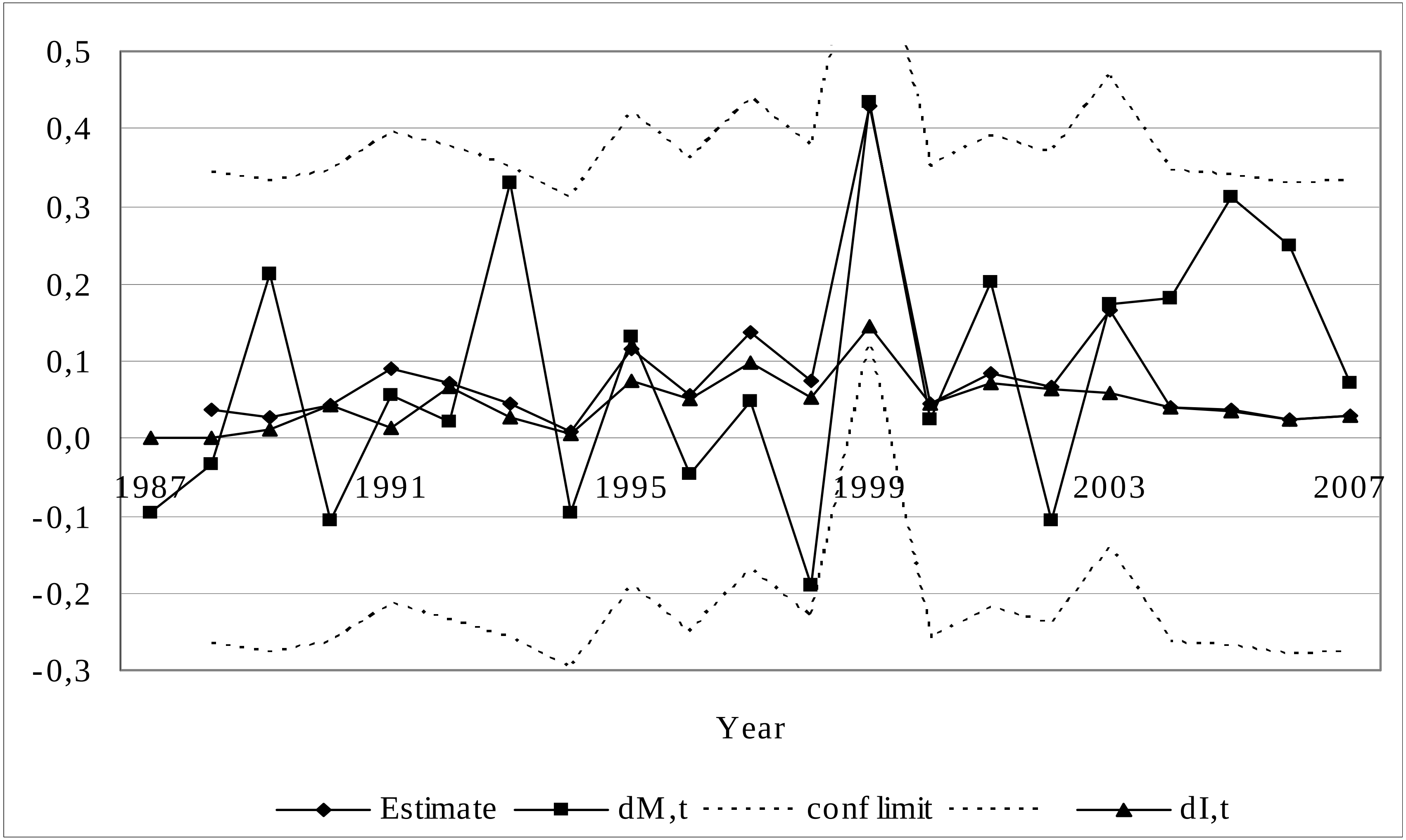
Parameterisation: regime-switching model: $\delta_{M:t}$ *vs.* $\delta_{I:t}$



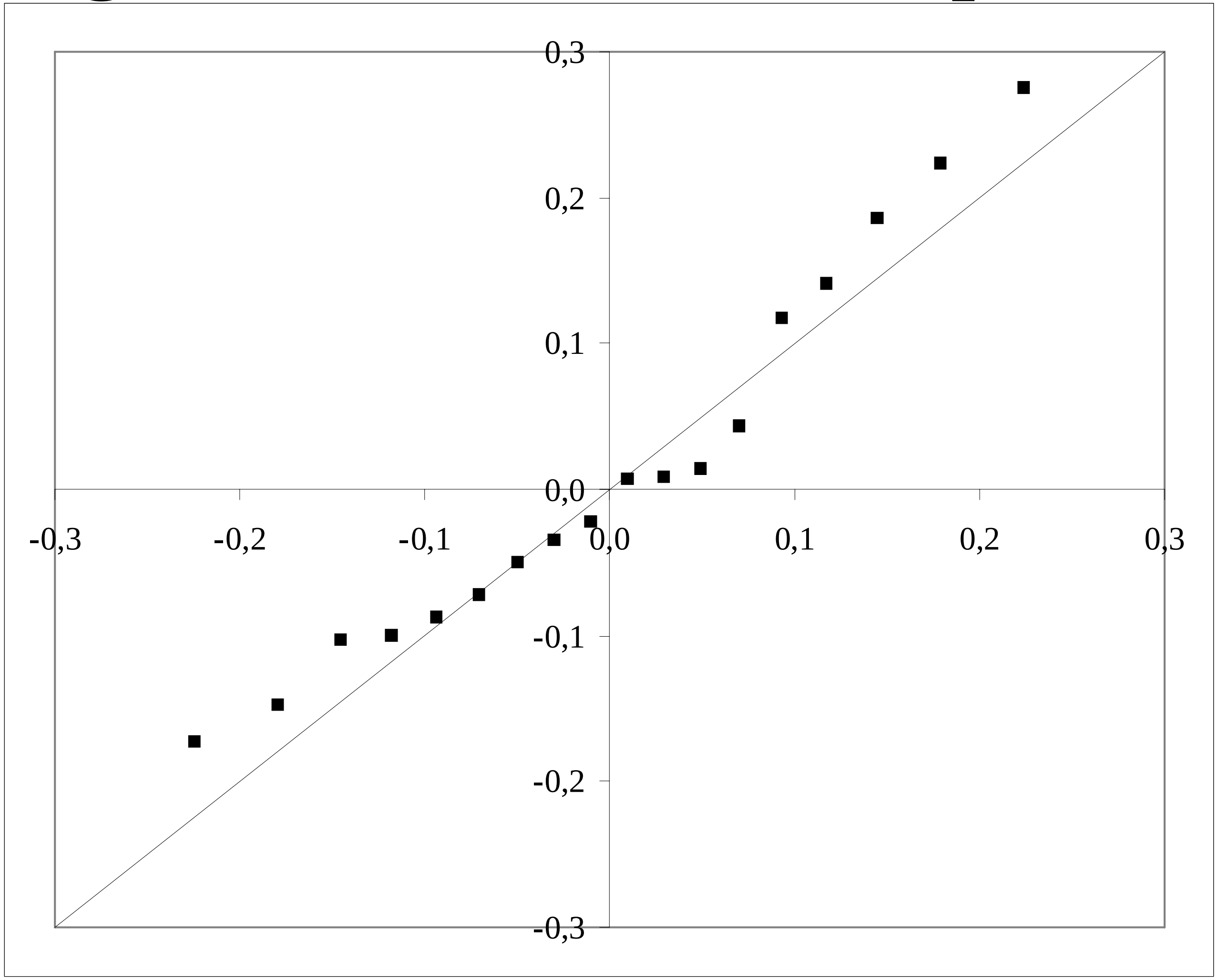
Parameterisation: exponential autoregressive model

Parameter	Details	
α	estimate	–13,75
	confidence limits	–16,9; 3,0
g	estimate	1
	confidence limits	1; 2,1
h	estimate	0,01
	confidence limits	0,01; 0,06
σ_M	estimate	0,155
	confidence limits	0,11; 0,22
k		2
l		10,34
A		–16,68
R		0,23
B		0,003

Parameterisation: exponential autoregressive model: time series



Parameterisation: exponential autoregressive model: Q-Q plot



Parameterisation: ARCH model

Parameter	Details	
g	estimate	1,76
	confidence limits	1; 2,9
h		0
a	estimate	0,025
	confidence limits	0,021; 0,029
b	estimate	0
	confidence limits	0; 0,04
k		2
l		9,30
A		–14,60
R		0,22
B		0,004

Summary

	Model		
Criterion	basic	regime-switching	exponential AR
A	–14,60	–12,50	–16,68
R	0,22	0,56	0,23
B	0,004	–0,051	0,003

Problems with the exponential AR model

- $g = 1$; $h = 0{,}01$
- spuriously good fit in 1999
- inaccuracy of adjustment for different period
- confidence limits of α: –16,9; 3,0
- allowance for ex-ante means exacerbate the problems

So rather use the basic model

Use of the model for predictive purposes

Per literature sources, ex-ante risk premium on equity = 0,037

This implies:

μ_M = 0,063 (not 0,084)

g = 1,39 (not 1,76)

Conclusion

$$\delta_{M;t} = g\delta_{I;t} + \sigma_M \varepsilon_t$$

where:

$$g = 1,39$$

$$\sigma_M = 0,159$$

Caveat

Lessons from the global financial crisis:

- a lower risk premium
- higher volatility
- a fatter downside tail

But:

- governments may bail out the financial institutions