ACTUARIAL SOCIETY OF SOUTH AFRICA

The Business of Change: 2010 and Beyond

A STOCHASTIC PROGRAMMING APPROACH TO INTEGRATED ASSET AND LIABILITY MANAGEMENT OF INSURANCE PRODUCTS WITH GUARANTEES

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Agenda

- Background
- Optimisation framework
- Results
- Conclusion

Background

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- Insurance products have become more complex by providing investors with various guarantees and bonus options
- This increase in complexity has provided an impetus for the investigation into integrated asset and liability management frameworks that could realistically address dynamic portfolio allocation in a risk-controlled way
- Examples
  - Yasuda-Kasai model by Cariño & Ziemba (1998)
  - Towers Perrin model by Mulvey & Thorlacius (1998)
  - CALM model of Consigli & Dempster (1998).
- More recent contributions
  - Dempster et al. (2006)
  - Consiglio et al. (2006)

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Background

- Multi-stage dynamic stochastic programming models has become a popular tool for integrated asset and liability modelling
  - Mean-variance (Markowitz, 1952) approach has a myopic view of managing investment risk over a single period
  - Dynamic stochastic optimisation provides the asset manager with an integrated way to model both assets and liabilities in a flexible manner
  - Takes into account multi-period dynamic asset allocation and the valuation of liabilities under future market conditions
  - Using this approach the rebalancing of the asset portfolio is modeled explicitly

Background

- Examples
  - Kouwenberg (2001)
  - Mulvey, Pauling & Madey (2003).
- Dempster et al. (2003)
  - Dynamic stochastic programming models will automatically hedge the current portfolio allocation against future uncertainties in asset returns and costs of liabilities over the analysis horizon.

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Background

- Dempster et al. (2006)
  - Proposed an asset and liability management framework and numerical results for a simple example of a closed-end guaranteed fund
  - Demonstrated the design of investment products with a guaranteed minimum rate of return focusing on the liability side of the product
  - Through back-testing they show that the proposed stochastic optimisation framework addresses the risk created by the guarantee in a reasonable way.
- Consiglio et al. (2006)
  - Study the same type of problem by structuring a portfolio for with-profit guarantee funds in the United Kingdom
  - The optimisation problem results in a non-linear optimisation problem
  - They demonstrated how the model can be used to analyse the alternatives to different bonus policies and reserving methods

Background

- We propose a multi-stage dynamic stochastic programming model for the integrated asset and liability management of insurance products with guarantees
  - Minimise the down-side risk of these products, as proposed in Dempster et al. (2006)
  - Our model also allows for portfolio rebalancing decisions over a multi-period horizon, as well as for flexible risk management decisions, such as the reinvestment of coupons at intermediate time steps.
  - We investigate with-profits guarantee funds as in Consiglio et al. (2006), by including regular bonus payments
  - Keep the optimisation problem linear, by changing the way bonuses are declared

Background

- We keep the problem linear, for two reasons
  - We can model the rebalancing of the portfolio at future decision times and by doing so the model automatically hedge the first stage portfolio allocation against projected future uncertainties in asset returns (see Dempster et al., 2003 and Dempster et al., 2006).
  - The model is flexible enough to take into account portfolio constraints such as the prohibition of short-selling, transaction costs and coupon payments.

Optimisation framework

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- Model features
  - As in Consiglio et al. (2006) we investigate the optimal asset allocation of with-profits guarantee funds, by including regular bonus payments
  - Once these bonuses have been declared, the bonus becomes guaranteed.
  - We also consider a proprietary company operating a fund on a 90/10 basis, i.e. the policyholder benefits in 90% of the asset share and the share holders 10%
  - It is assumed that no policyholder contributions are allowed after the initial upfront premium
  - The time horizon of the fund is T years and the minimum guaranteed rate of return is g on the initial wealth
  - We use six different assets, namely, (semi-annual) coupon bearing bonds with maturities 5, 7, 10, 15 and 19 years and the FTSE/JSE Top 40 equity index.

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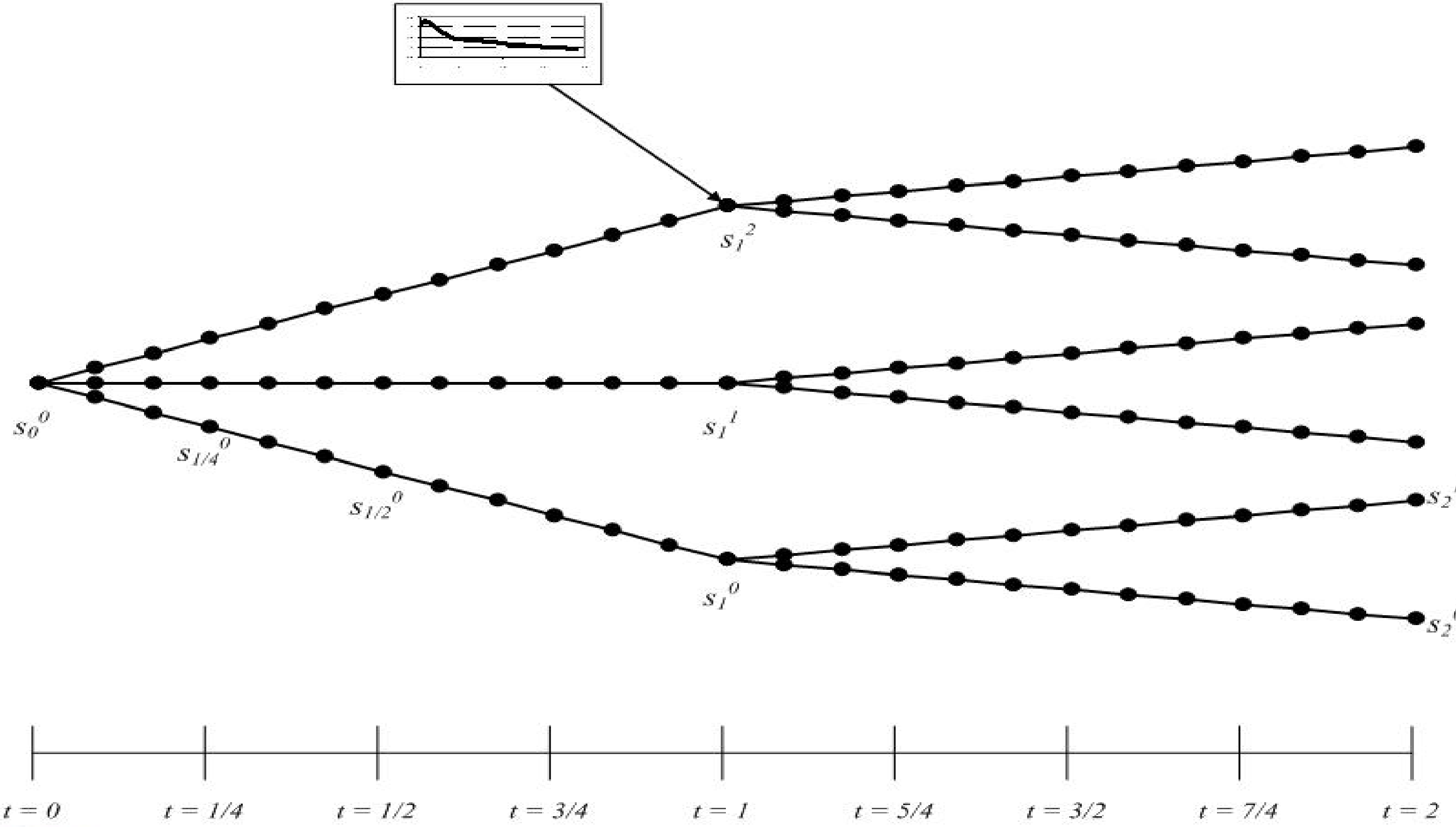
Optimisation framework

- Model features
  - Scenario representation



Optimisation framework

- Variables and parameters
  - Time sets

$T^{total} = \left\{0, \frac{1}{12}, \frac{2}{12}, \ldots, T\right\}$: set of all times considered in the stochastic program;

$T^{d} = \left\{0, 1, 2, \ldots, T-1\right\}$: set of decision times;

$T^{i} = T^{total} \setminus T^{d}$: set of intermediate times;

$T^{c} = \left\{\frac{1}{2}, \frac{3}{2}, \ldots, T-\frac{1}{2}\right\}$: set of coupon payment time between decision times;

Note that $T^{d} \cap T^{i} = 0$ and $T^{c} \subset T^{i}$.

Optimisation framework

- Variables and parameters
  - Index sets

$\Sigma_t = \{ s_t^v \mid v = 1, 2, \dots, S_t \}$: set of stages at period t;

SI : set of stock indices;

$B = \{ B_\tau \}$: set of government bonds with maturity τ;

$I = SI \bigcup B$: set of all instruments;

Optimisation framework

- Variables and parameters
  - Parameters

δ_{B_τ} : coupon rate of a government bond with maturity τ ;

F_{B_τ} : face value of a government bond with maturity τ ;

$r^s_{t,\tau}$: zero-rate with maturity τ at period t at stage s ;

g : minimum guaranteed rate of return;

ρ : regulatory equity to debt ratio;

$r^s_{t,b}$: benchmark rate at period t at stage s ;

γ : policyholders' rate of participation in the profits of the firm;

β : target terminal bonus;

$P^{a,s}_{t,i}$ / $P^{b,s}_{t,i}$: ask or bid price of asset $i \in I$ at period t at stage s ;

f_a / f_b : proportional transaction costs on ask or bid transactions;

p^s_t : probability of stage s at period t ;

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Optimisation framework

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- Variables and parameters
  - Decision variables

$x_t^s = \{x_{t,i}^s\}_{i \in I}$: quantities of assets bought at period t at stage s;

$y_t^s = \{y_{t,i}^s\}_{i \in I}$: quantities of assets sold at period t at stage s;

$z_t^s = \{z_{t,i}^s\}_{i \in I}$: quantities of assets hold at period t at stage s;

A_t^s : value of assets account at period t at stage s;

L_t^s : value of liability account at period t at stage s;

E_t^s : value of equity account at period t at stage s;

c_t^s : amount of equity provided by shareholders at period t at stage s;

SF_t^s : amount of shortfall at period t at stage s;

RB_t^s : regular bonus payment declared at period t at stage s;

TB_T^s : policyholders terminal bonus at period T at stage s;

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Optimisation framework

- Variable dynamics and constraints
  - Cash balance constraints

$$\sum_{i \in I} P_{0,i}^{a,s} x_{0,i}^{s} \left(1 + f_a\right) = A_0^s \text{ , for } t \in \{0\} \text{ and } s \in \Sigma_t \text{ ,}$$

$$\sum_{i \in I} P_{t,i}^{b,s} y_{t,i}^{s} \left(1 - f_b\right) + \sum_{i \in I \setminus \{SI\}} \tfrac{1}{2} \delta_i F_i y_{t,i}^{s} + c_t^s = \sum_{i \in I} P_{t,i}^{a,s} x_{t,i}^{s} \left(1 + f_a\right) \text{, for}$$

$$t \in T^d \setminus \{0\} \text{, and } s \in \Sigma_t \text{ .}$$

  - Short sale constraints

$$x_{t,i}^s \geq 0 \text{, for all } i \in I \text{, } t \in T^{total} \setminus \{T\} \text{ and } s \in \Sigma_t$$

$$y_{t,i}^s \geq 0 \text{, for all } i \in I \text{, } t \in T^{total} \setminus \{0\} \text{ and } s \in \Sigma_t$$

$$z_{t,i}^s \geq 0 \text{, for all } i \in I \text{, } t \in T^{total} \setminus \{T\} \text{ and } s \in \Sigma_t$$

Optimisation framework

- Variable dynamics and constraints
  - Inventory constraints

$$z_{0,i}^{s} = x_{0,i}^{s}\text{, for } t \in \{0\} \text{ and } s \in \Sigma_t\text{,}$$

$$z_{t,i}^{s} = z_{t,i}^{s-} + x_{t,i}^{s} - y_{t,i}^{s}\text{, for } i \in I\text{, } t \in T^{total} \setminus \{0\} \text{ and } s \in \Sigma_t$$

  - Information constraints

$$x_{t,i}^{s} = y_{t,i}^{s} = 0\text{, for } i \in I\text{, } t \in T^{i} \setminus T^{c} \text{ and } s \in \Sigma_t$$

  - Coupon reinvestment constraints

$$x_{t,i}^{s} = \frac{\frac{1}{2}\delta_i F_i z_{t,i}^{s-}}{P_{t,i}^{a,s}\left(1+f_a\right)}\text{, for } i \in I \setminus \{SI\}\text{, } t \in T^{c} \text{ and } s \in \Sigma_t$$

$$y_{t,i}^{s} = 0\text{, for } i \in I \setminus \{SI\}\text{, for } t \in T^{c} \text{ and } s \in \Sigma_t\text{,}$$

$$x_{t,SI}^{s} = 0,\ y_{t,SI}^{s} = 0\text{, for } t \in T^{c} \text{ and } s \in \Sigma_t\text{.}$$

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Optimisation framework

- Variable dynamics and constraints
  - Asset account

$$A_0^s = L_0 + E_0^s \text{, for } t \in \{0\} \text{ and } s \in \Sigma_t \text{,}$$

$$A_t^s = \sum_{i \in I} P_{t,i}^{a,s} z_{t,i}^s \left(1 + f_a\right) \text{, for } t \in T^{total} \setminus \{T\} \text{ and } s \in \Sigma_t \text{,}$$

$$A_T^s = \sum_{i \in I} P_{T,i}^{b,s} z_{T-1/12,i}^{s-} \left(1 - f_b\right) + \sum_{i \in I \setminus \{SI\}} \tfrac{1}{2} \delta_i F_i z_{T-1/12,i}^{s-} \text{, for } s \in \Sigma_T \text{.}$$

  - Liability account

$$L_0^s = L_0 \text{, for } t \in \{0\} \text{ and } s \in \Sigma_t \text{,}$$

$$L_t^s = L_{t-1/12}^{s-} e^{1/12 g} + RB_t^s \text{, for } t \in T^{total} \setminus \{T\} \text{ and } s \in \Sigma_t$$

  - Equity account

$$E_0^s = c_0^s \text{, for } t \in \{0\} \text{ and } s \in \Sigma_t \text{,}$$

$$E_t^s = E_{t-1/12}^{s-} e^{r_{t-1/12,t}^{s-}} + c_t^s \text{, for } t \in T^{total} \setminus \{T\} \text{ and } s \in \Sigma_t$$

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Optimisation framework

- Variable dynamics and constraints
  - Regular bonus constraints
  - We follow the approach described by Consiglio et al. (2006) which is based on that of Ross (1989) where the regular bonuses are determined by aiming for a target terminal bonus
  - Assumed to be declared at decision times only (i.e. annually).
  - Assumed that the asset account will grow at the benchmark rate

$$A_T^s = A_t^{bs} e^{r_{t,b}^s (T-t)}$$

$$A_t^{bs} = \sum_{i \in I} P_{t,i}^{b,s} z_{t-1/12,i}^{s-} \left(1 - f_b\right) + \sum_{i \in I \setminus \{SI\}} \tfrac{1}{2} \delta_i F_i z_{t-1/12,i}^{s-}$$

  - Assumed that the liabilities will grow at the minimum growth rate and that the regular bonus payment that is declared will stay constant for the remainder of the time

$$L_T^s = L_{t-1/12}^{s-} e^{g(T-t+1/12)} + RB_t^s \boxed{\left(\frac{e^{g(T-t)} - 1}{e^g - 1} + e^{g(T-t)} \right)}$$

  - The terminal bonus

$$TB_T^s = \gamma \left(A_T^s - L_T^s \right)$$

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Optimisation framework

- Variable dynamics and constraints
  - Regular bonus constraints
  - The terminal bonus received by the policyholders need to constitute β% of the total amount received by the policyholders

$$\frac{TB_T^s}{TB_T^s + L_T^s} = \beta$$

  - Solving

$$RB_t^s = \frac{\gamma(1-\beta) A_t^{b,s} e^{r_{t,b}^s (T-t)} - \left(\beta + \gamma(1-\beta)\right) L_{t-\frac{1}{12}}^{s-} e^{g\left(T-t+\frac{1}{12}\right)}}{\left(\beta+\gamma(1-\beta)\right)\left(\dfrac{e^{g(T-t)}-1}{e^{g}-1} + e^{g(T-t)}\right)}$$

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Optimisation framework

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- Variable dynamics and constraints
  - Regular bonus constraints
  - When the expected terminal asset amount exceeds the expected terminal liability amount regular bonuses will increase.
  - When the expected terminal liability amount exceeds the expected terminal asset amount the regular bonus will be negative
  - As this will be unfair towards policyholders to declare negative bonuses the constraint

$$RB_t^s >= \frac{\gamma(1-\beta)A_t^{b,s}e^{r_{t,b}^s(T-t)} - \left(\beta+\gamma(1-\beta)\right)L_{t-\frac{1}{12}}^{s-}e^{g\left(T-t+\frac{1}{12}\right)}}{\left(\beta+\gamma(1-\beta)\right)\left(\dfrac{e^{g(T-t)}-1}{e^{g}-1}+e^{g(T-t)}\right)}, \text{ for}$$

$$t \in \left(T^d \setminus \{0\}\right) \cup \{T\}, \text{ and } s \in \Sigma_t,$$

  - Where

$$RB_t^s >= 0$$

$$RB_t^s = 0 \quad \text{for} \quad t \in \left(T^i \cup \{0\}\right) \setminus \{T\}, \text{ and } \quad s \in \Sigma_t$$

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Optimisation framework

- Variable dynamics and constraints
  - Regular bonus constraints
  - Consiglio et al. (2006) also consider the *working party approach* based on Chadburn (1997) which is based on work done by Institute of Actuaries Working Party.
  - This approach declares regular bonuses (in return form) to reflect the benchmark return subject to the liability account remaining lower than the value of the *reduced asset account*, where the reduced assets accumulates at 75% of the return on assets.
  - Consiglio et al. (2006) test their model with both these features and find that bonus policies based on aiming for a target terminal bonus outperforms bonus polices based on the working party approach.

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Optimisation framework

- Variable dynamics and constraints
  - Shortfall constraints
  - Determine the shortfall of the portfolio at each stage for each time period

$$SF_0^s + L_0 >= (1+\rho) L_0^s \text{, for } t \in \{0\} \text{ and } s \in \Sigma_t$$

$$SF_t^s + A_t^{bs} >= (1+\rho) L_{t-1/12}^s e^{g\,1/12} \text{, for}$$

$$t \in \left(T^{total} \setminus \{0\}\right) \text{, and } s \in \Sigma_t$$

$$A_t^{bs} = \sum_{i \in I} P_{t,i}^{b,s} z_{t-1/12,i}^{s-} (1 - f_b) + \sum_{i \in I \setminus \{SI\}} \tfrac{1}{2} \delta_i F_i z_{t-1/12,i}^{s-}$$

$$SF_t^s >= 0 \text{ for } t \in T^{total} \text{, and } s \in \Sigma_t$$

  - The shortfall at each decision period is funded by the shareholders

$$c_t^s = SF_t^s \text{ for } t \in \left(T^d\right) \cup \{T\} \text{ and } s \in \Sigma_t$$

$$c_t^s = 0 \text{ for } t \in T^i \setminus \{T\} \text{, and } s \in \Sigma_t$$

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Optimisation framework

- Variable dynamics and constraints
  - Objective function
  - Maximising the expected excess wealth of the shareholders and the minimising the average expected shortfall over the decision periods

$$\max_{\left\{\substack{x_{t,i}^s, y_{t,i}^s, z_{t,i}^s : \\ i \in I, t \in T^d \cup \{T\}, s \in \Sigma_t}\right\}} \left\{ \begin{array}{c} (1-\alpha) \sum_{s \in \Sigma_T} p^s (1-\gamma)\left(\left(A_T^s - L_T^s\right) - E_T^s\right) \\ -\alpha \sum_{t \in T^{total}} \sum_{s \in \Sigma_t} p^s \frac{SF_t^s}{\left|T^{total}\right|} \end{array} \right\}$$

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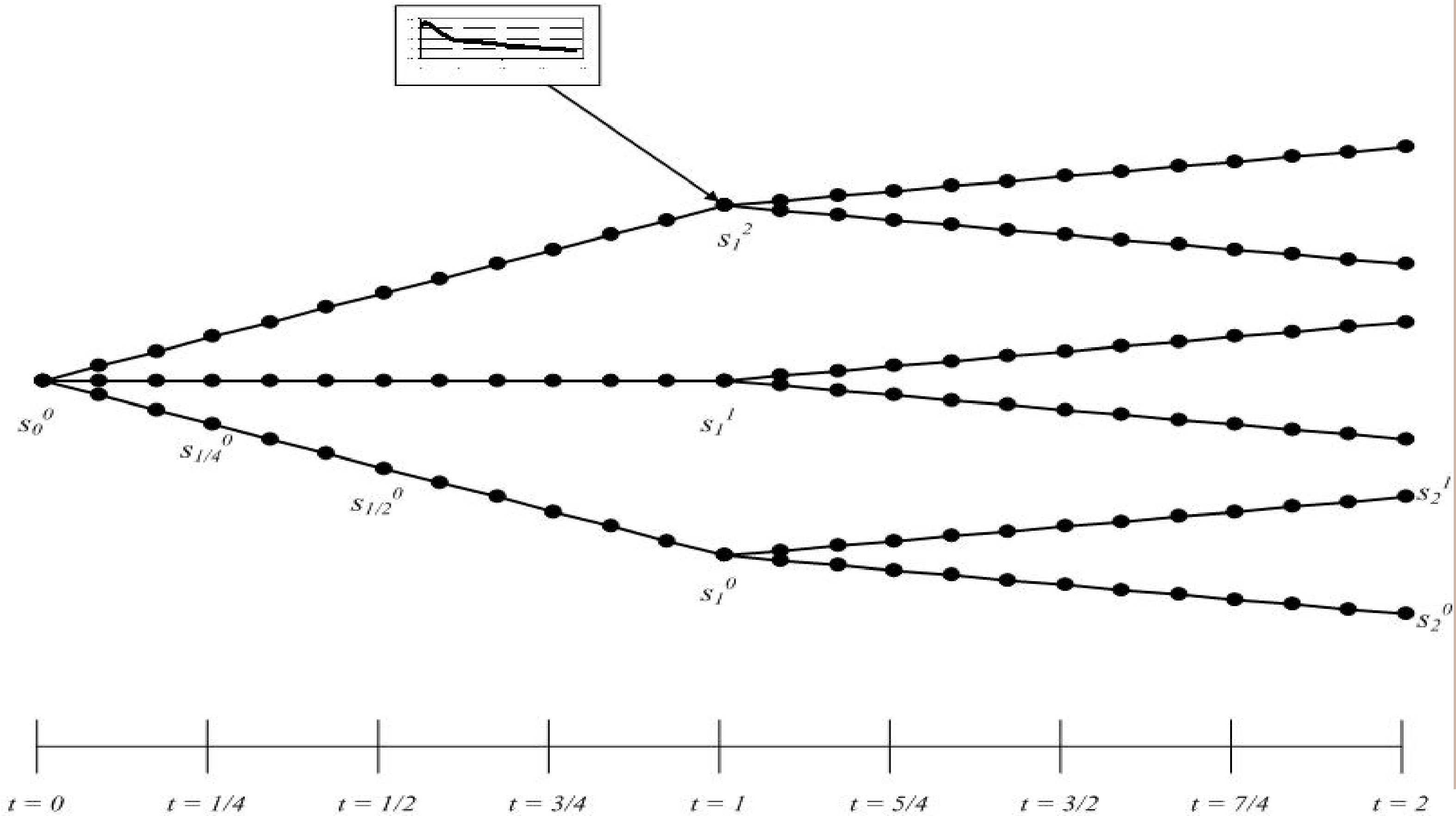
Results

- Scenarios
  - Scenario representation



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Results

- Scenarios
  - Four-factor yield curve representation – Svensson (1994)
  - Macro-economic variables
    - Manufacturing capacity utilisation
    - Inflation
    - Repo-rate
  - Kalman filter - parameter estimation
  - Parallel simulations and clustering approach
  - Arbitrage – excluded ex post

Results

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- Data and instruments
  - Five semi-annual coupon bearing bonds with maturities 5, 7, 10, 15 and 19 years
  - FTSE/JSE Top 40 equity index
  - BESA Perfect Fit Bond Curves
  - End of month data August 1999 - April 2008
  - Tree-string

Year	Tree-string
April 03	5.5.5.5.5 = 3125
April 04	8.8.8.8 = 4096
April 05	15.15.15 = 3375
April 06	56.56 = 3136
April 07	3125

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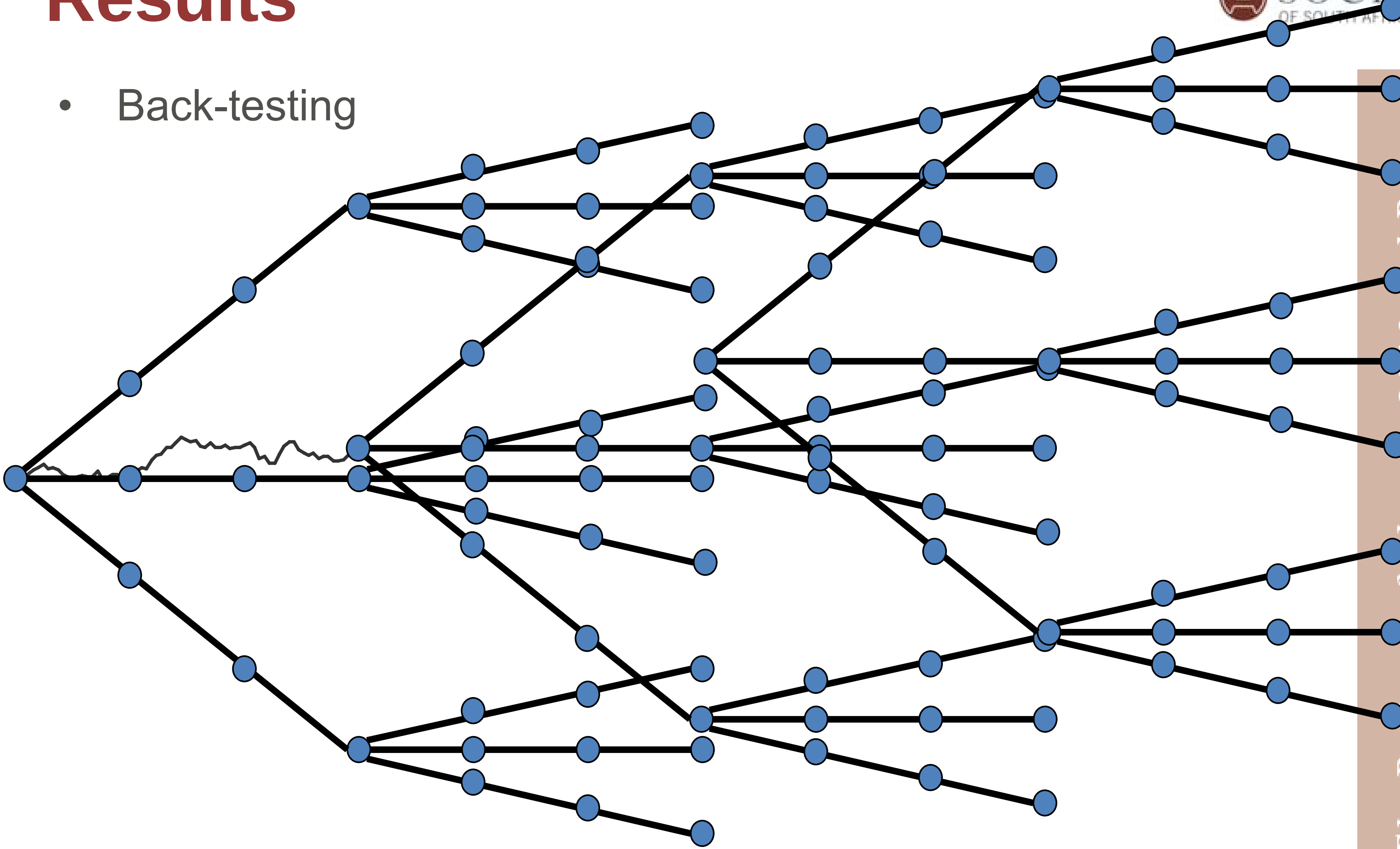
Results

- Back-testing



Results

- Back-testing
  - We back-test the objective function over a period of five years, from April 2003 through to April 2008
  - Different levels of minimum guarantee
  - Different levels of risk-aversion
  - SAS\OR (PROC OPTMODEL)
  - Annual excess return on equity (ExROE)

$$\sqrt[T]{\frac{(1-\gamma)(A_T - L_T) + \gamma E_T}{E_T}} - 1$$

  - Cost of the guarantee taken to be the expected present value of the final equity deducting the regulatory equity or equity at the start

$$\left(\frac{E_T}{\prod_{t=1}^{T} \left(1 + r_{t,t+1/12}\right)} - E_0 \right)$$

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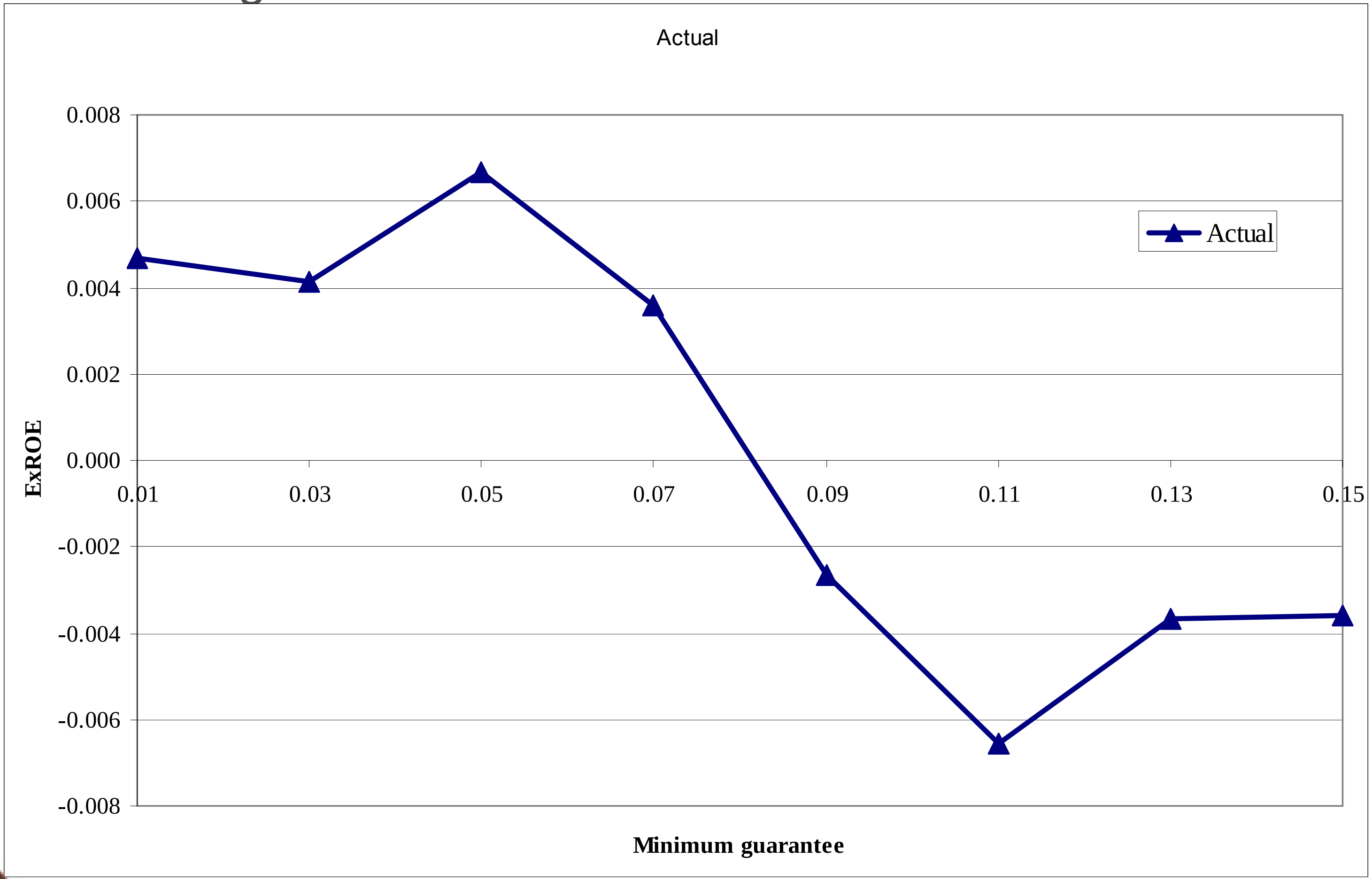
Results

- Shareholders annual excess return on equity for different levels of minimum guarantee



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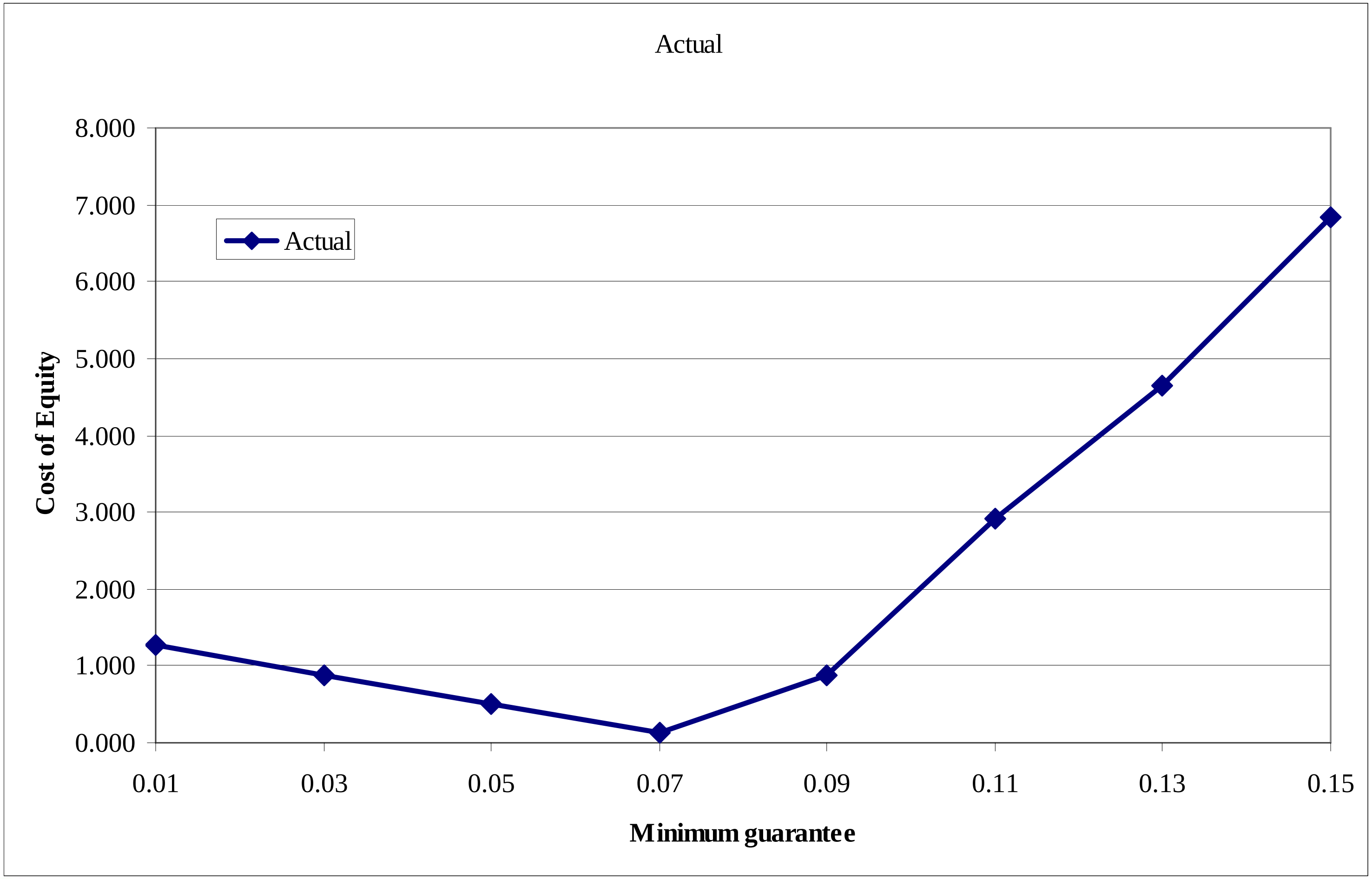
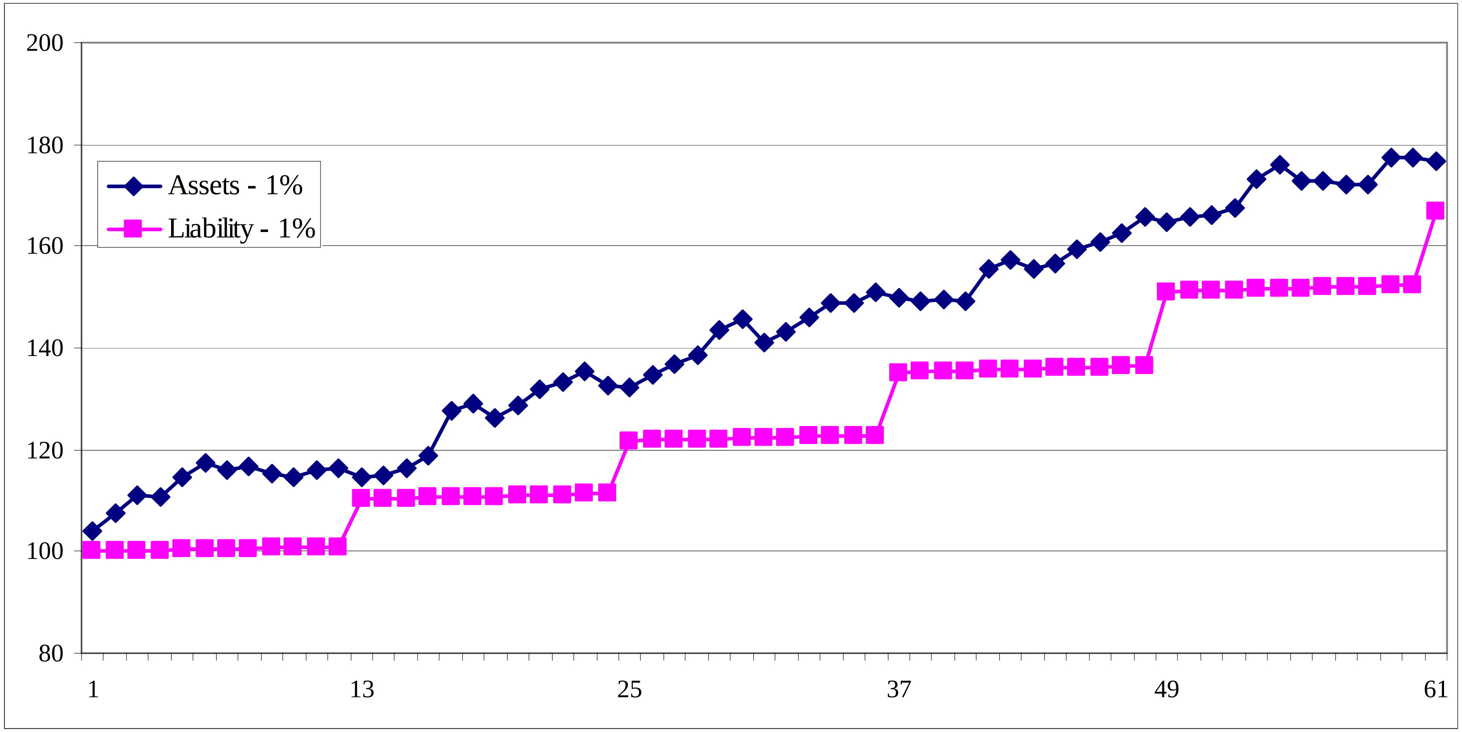
Results

- Cost of equity for different levels of minimum guarantee



Results

- Asset and Liability account at 1%, 9% and 15% minimum guarantee



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Results

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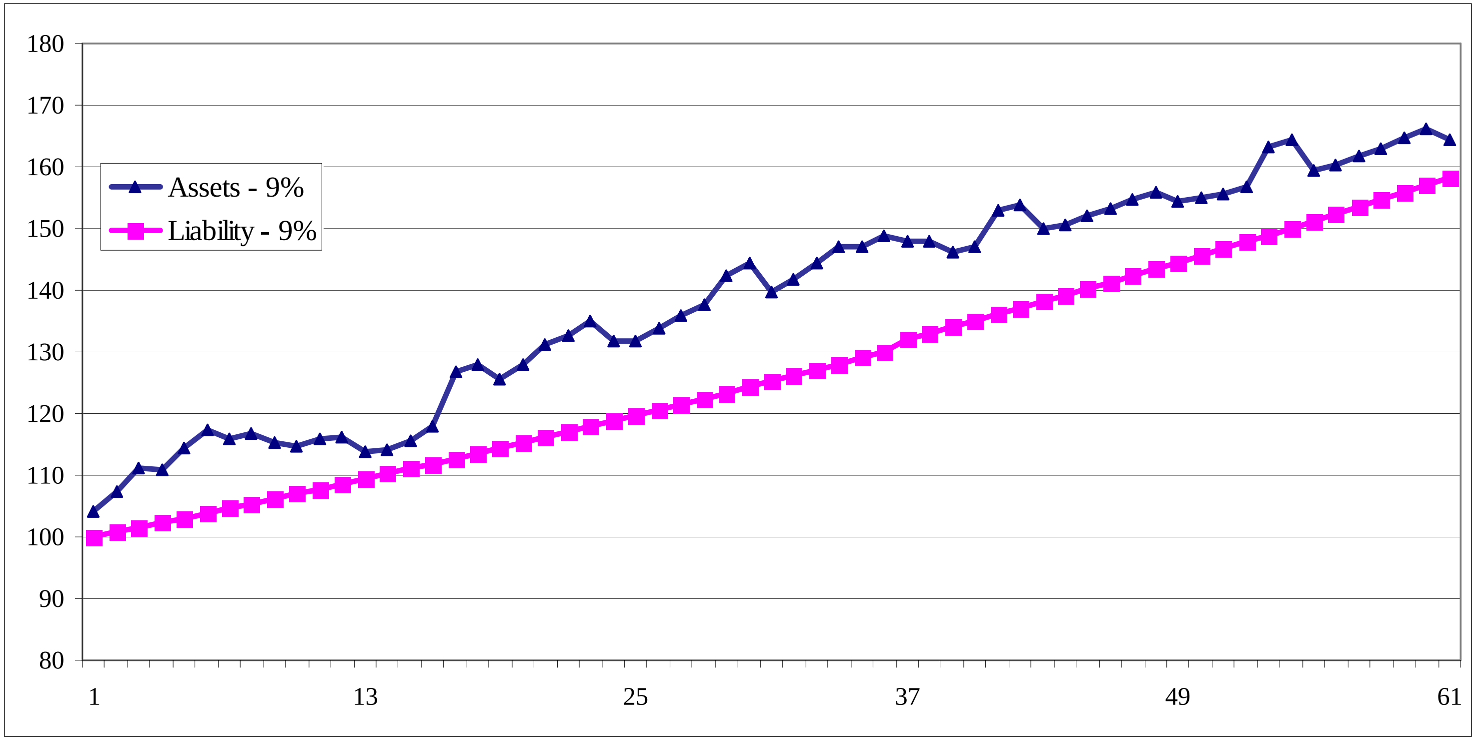
- Asset and Liability account at 1%, 9% and 15% minimum guarantee



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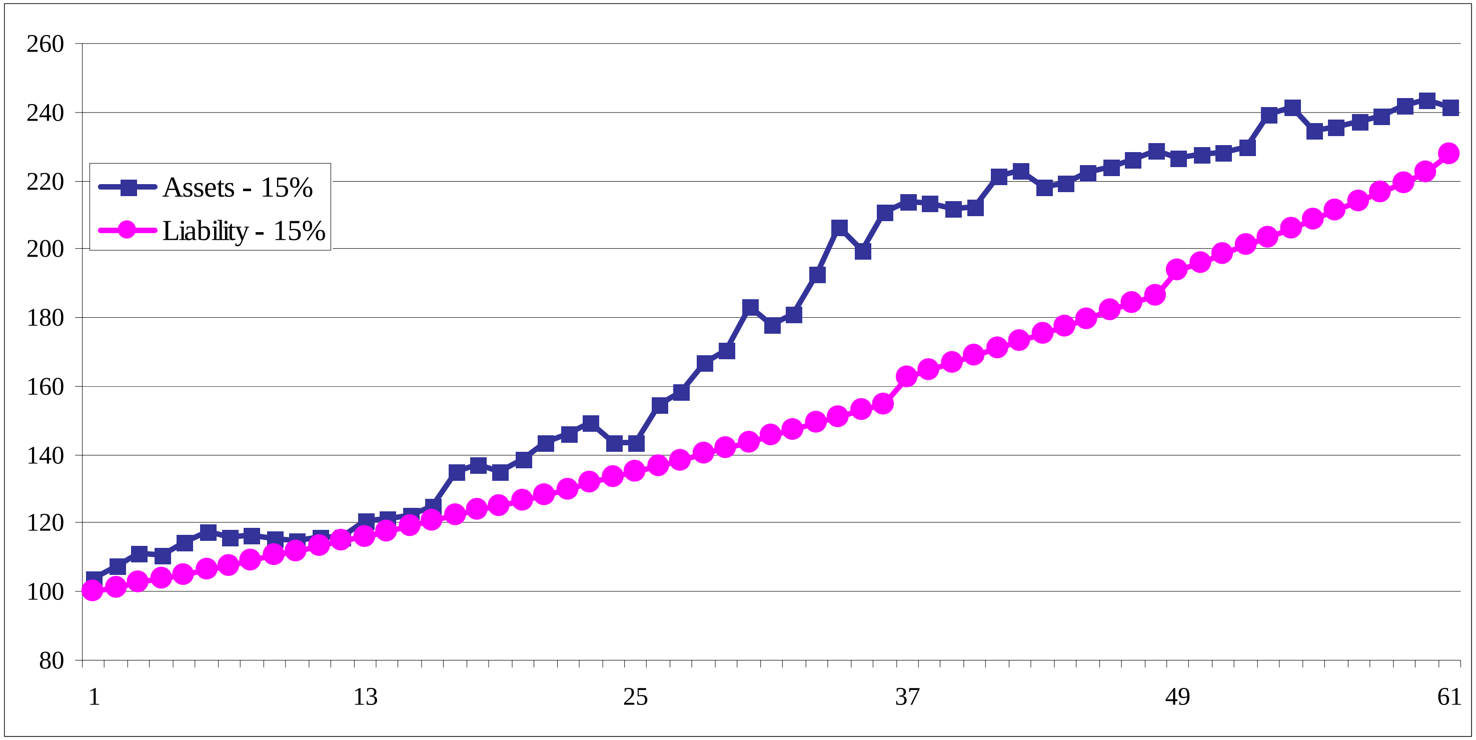
Results

- Asset and Liability account at 1%, 9% and 15% minimum guarantee



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Results

- Liabilities with different bonus options
  - Consiglio et al. (2006) specify regular bonuses in return form.
  - More realistic than our formulation of discrete annual payments which we define in order to keep the problem linear.
  - Consiglio et al. (2006) assumes that the bonus return that is declared at decision times will stay constant through out the remainder of the term giving the terminal liability value as

$$L_T^s = L_t^s e^{g(T-t)} e^{RB_t^s (T-t)}$$

  - With all other assumptions staying constant the regular bonus yields:

$$RB_t^s = \max\left[\frac{1}{(T-t)} \ln\left(\frac{\gamma(1-\beta) A_t^{b,s} e^{r_{t,b}^s (T-t)}}{(\beta+\gamma(1-\beta)) L_t^s e^{g(T-t)}}\right), 0\right]$$

Results

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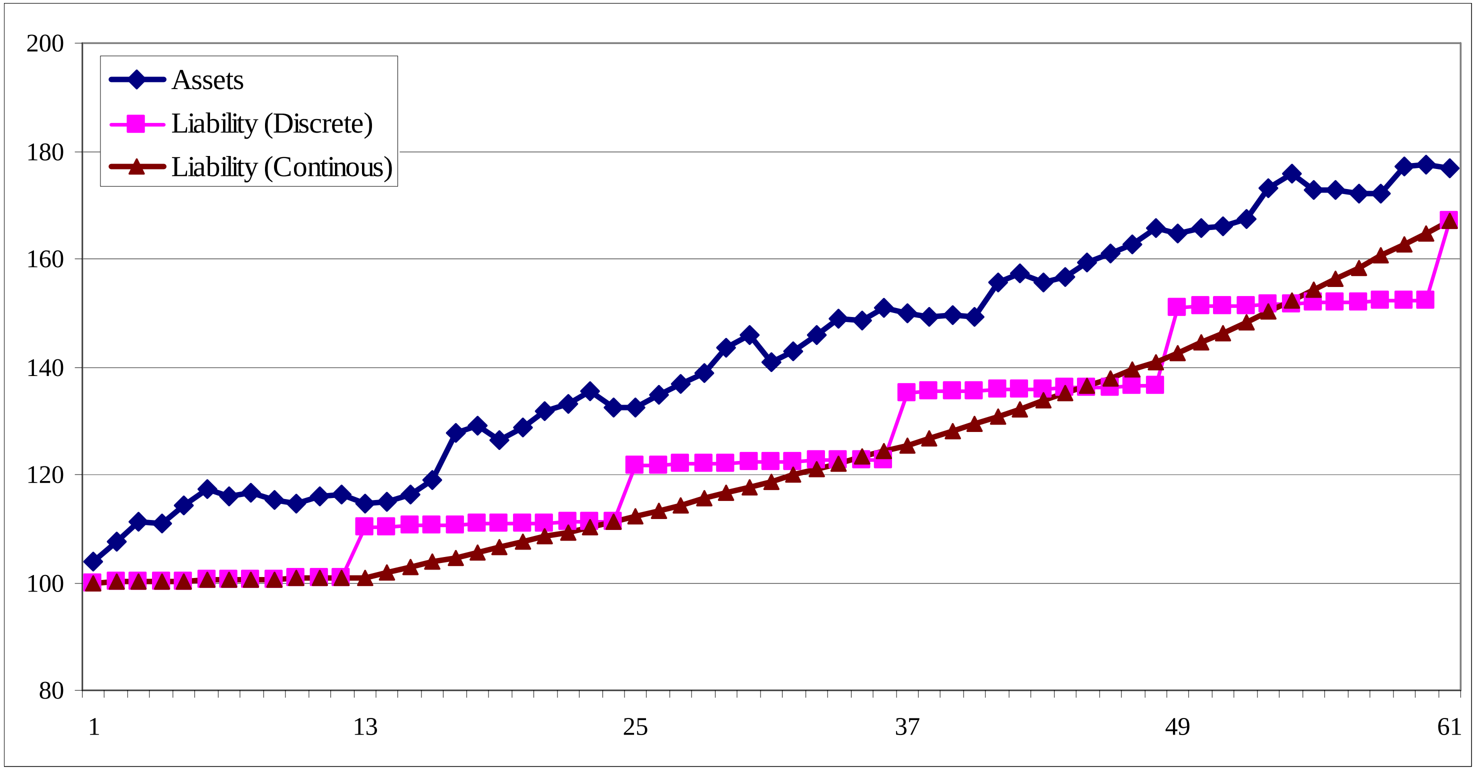
- Liabilities with different bonus options at 1% minimum guarantee



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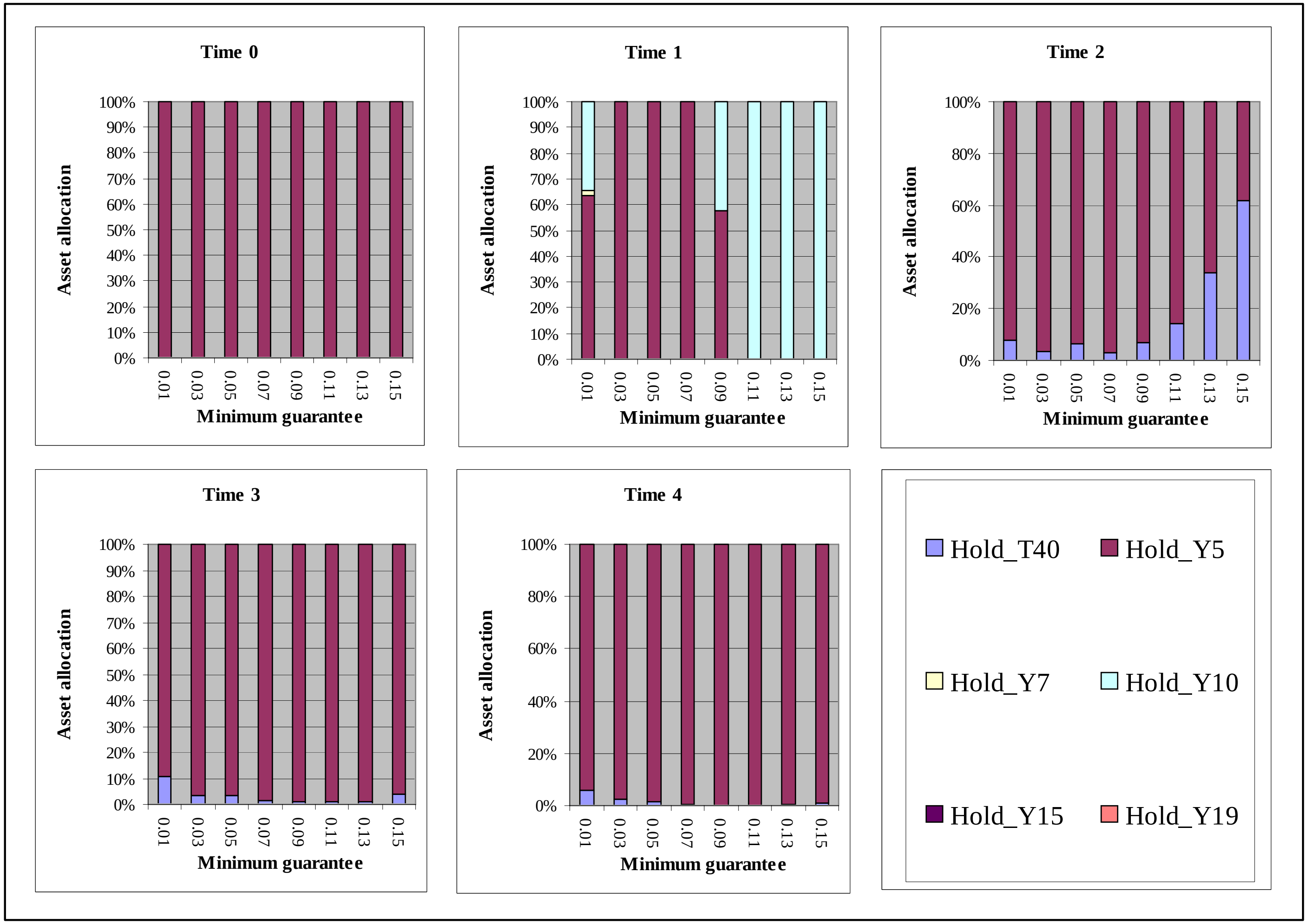
Results

- Asset allocation for different levels of minimum guarantee



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Results

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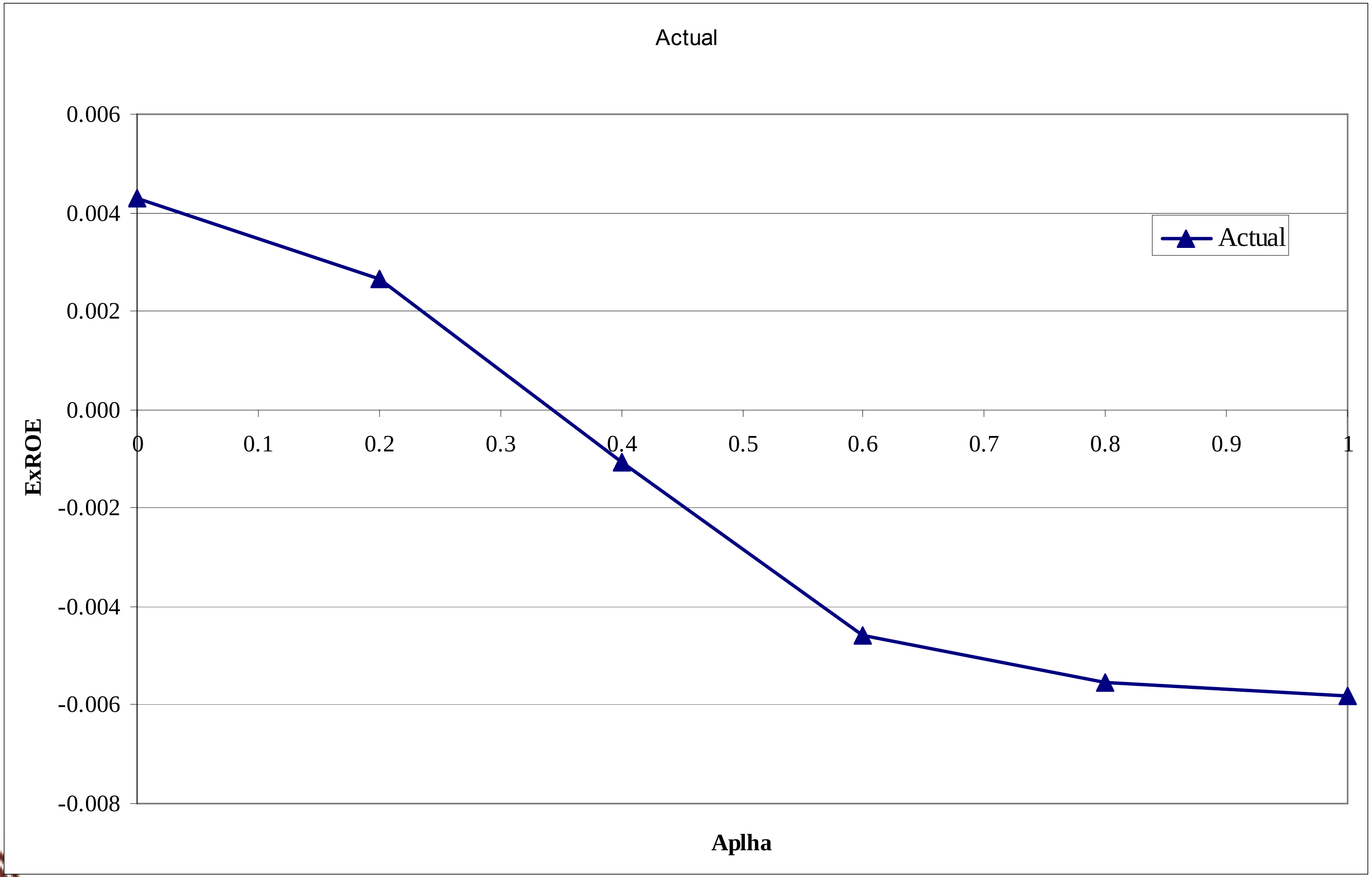
- Shareholders annual excess return on equity for different levels of risk-aversion at 9% minimum guarantee



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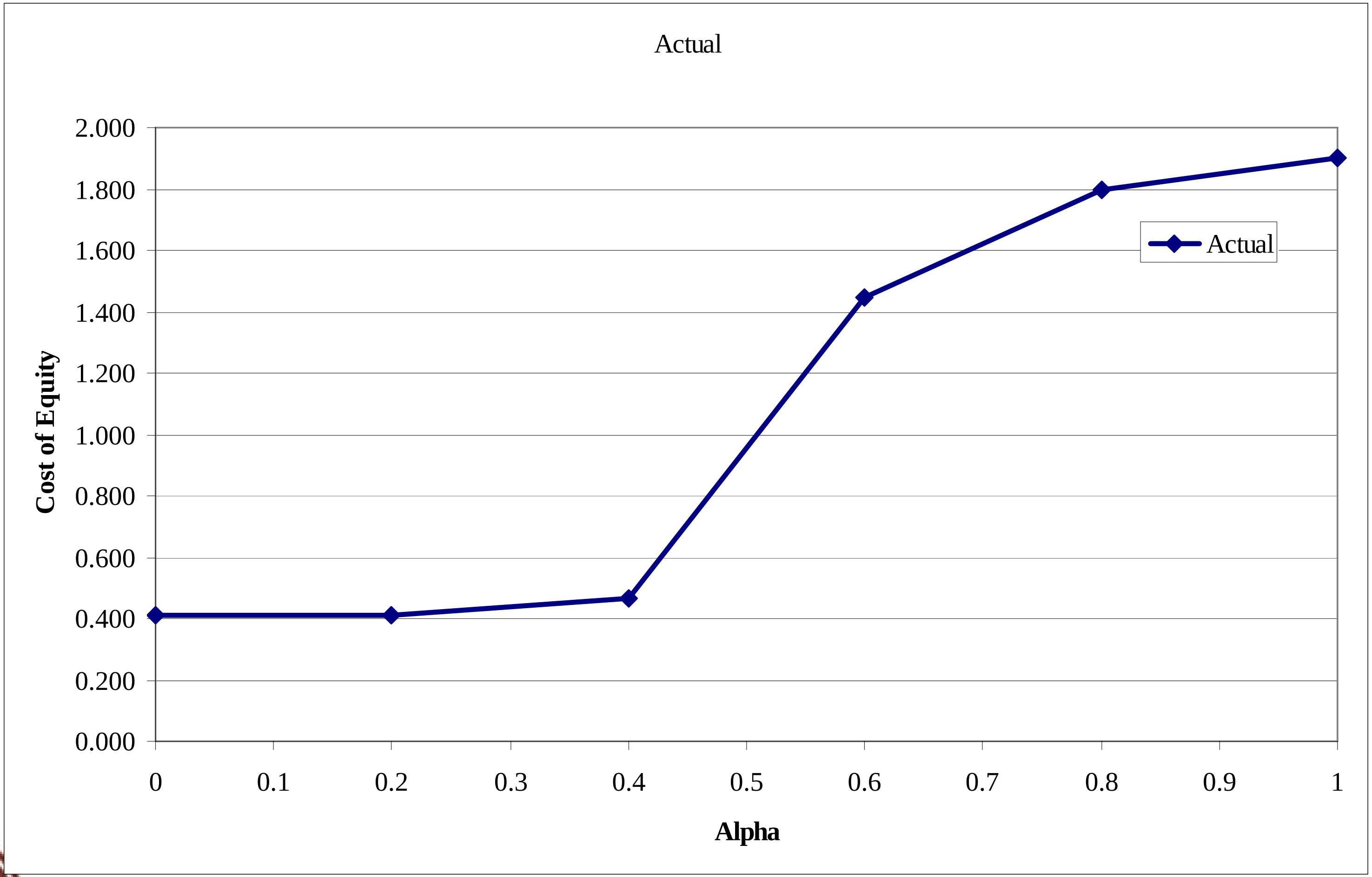
Results

- Cost of equity for different levels of risk-aversion at 9% minimum guarantee



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Results

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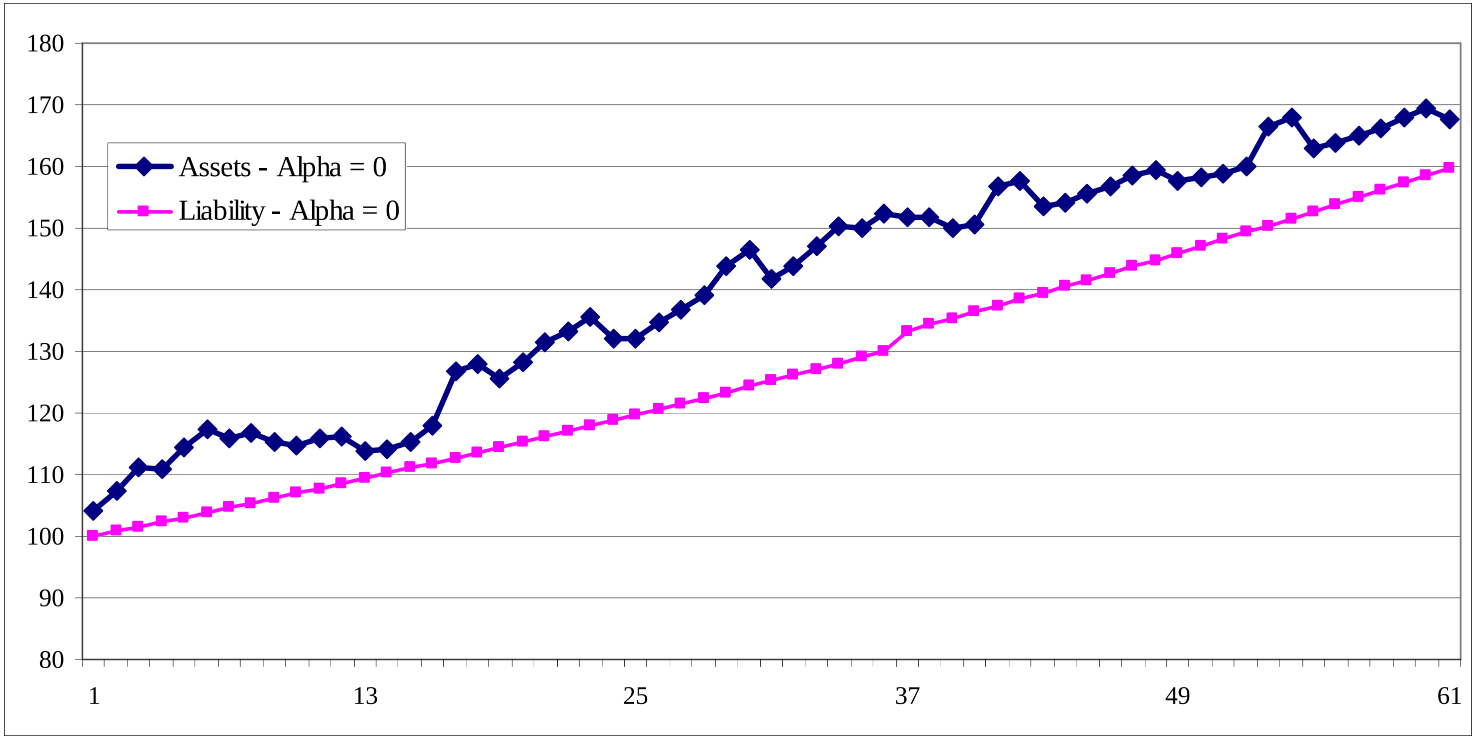
- Asset and Liability account at 0 and 1 level of risk-aversion for 9% minimum guarantee



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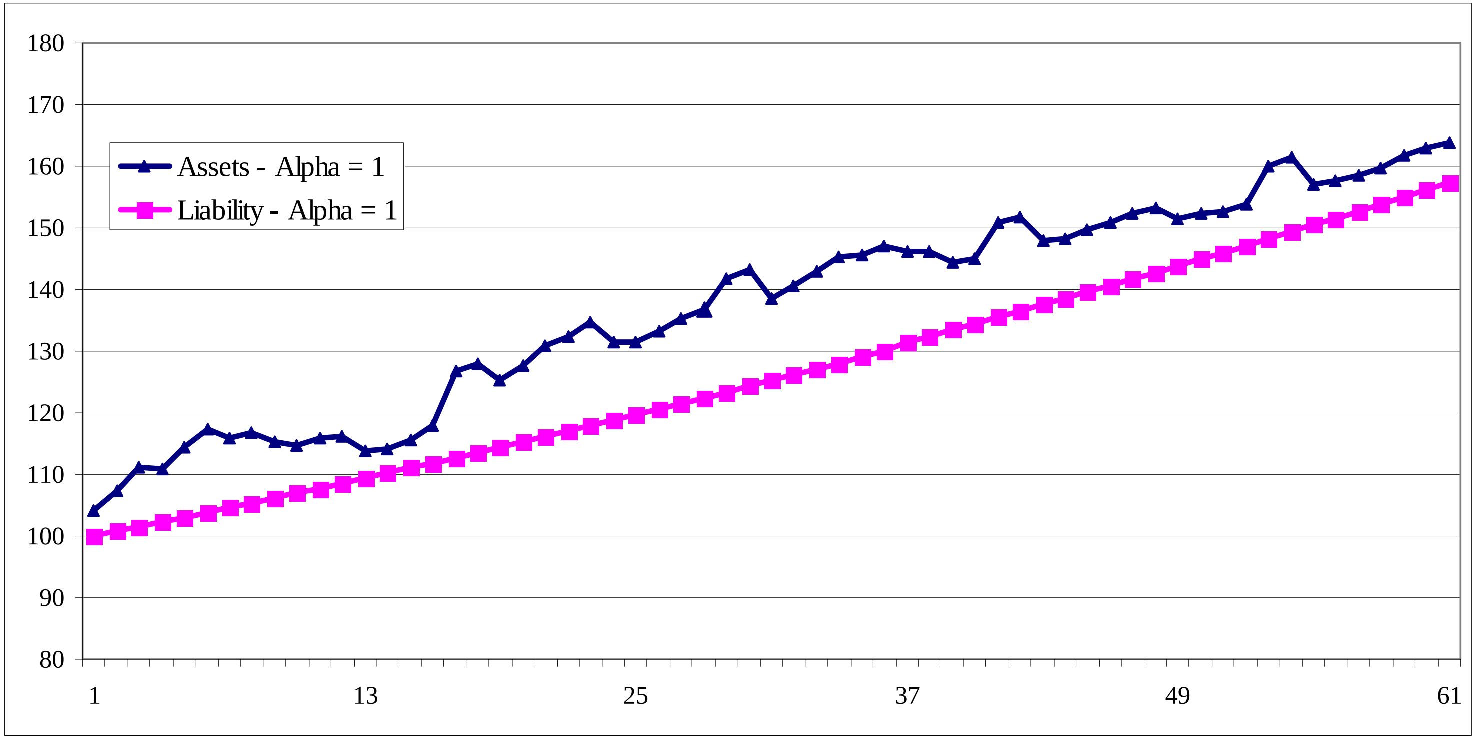
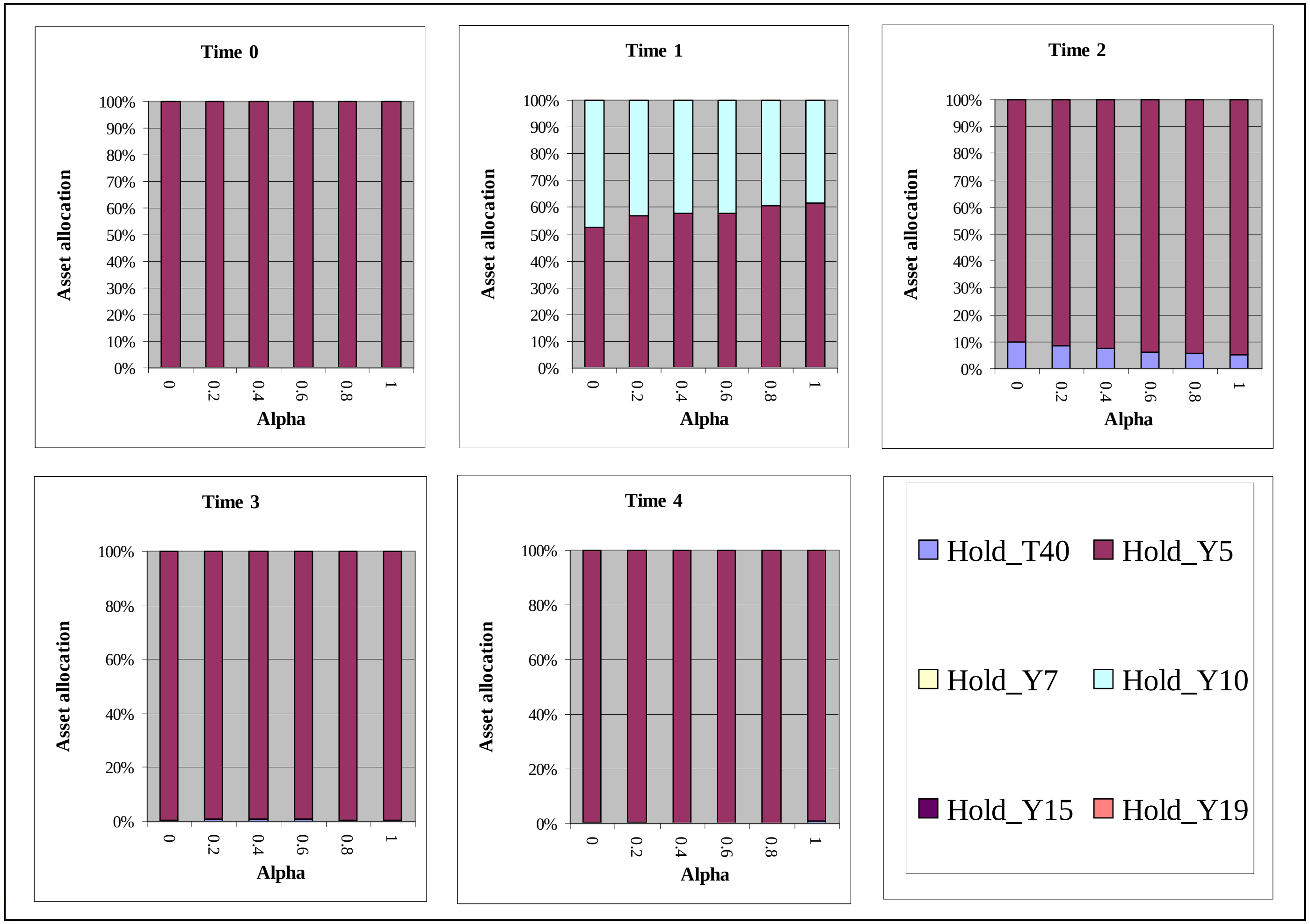
Results

- Asset and Liability account at 0 and 1 level of risk-aversion for 9% minimum guarantee



Results

- Asset allocation different levels of risk-aversion at 9% minimum guarantee



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Conclusion

- Presented a multi-stage dynamic stochastic programming model for the integrated asset and liability management of insurance products with guarantees
- Included regular bonus payments and kept the optimisation problem linear
- Shown the model features at different levels of minimum guarantee and different levels of risk aversion
- As Consiglio et al. (2006) have shown, the model can also be used for analysing the investment decision made by the insurance firm.
- Future extensions may look at the inclusion of other policy features.

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