A COMPARISON OF ANNUITISATION OPTIONS UNDER RUIN THEORY AND DISCOUNTED UTILITY MODELS

By Megan Butler, Brian Hu and Dwayne Kloppers

Presented at the Actuarial Society of South Africa's 2012 Convention
16–17 October 2012, Cape Town International Convention Centre

ABSTRACT

Individuals in defined contribution retirement funds currently have a number of options as to how to finance their post-retirement spending. The paper considers the ranking of selected annuitisation strategies when using both ruin theory and discounted utility models. The ruin theory model, where ruin is defined as income falling below a given threshold, favours living annuities with high equity components if there is not sufficient capital at retirement to purchase an inflation-linked annuity. The discounted utility models, which explicitly allow for bequests, produce a range of preferred strategies, depending on the base pension against which preferences are measured. For the ruin theory model and one discounted utility model, the results were very sensitive to the full accumulated wealth, which limits the extent to which the process can be automated or outsourced. Given the lack of definitive evidence to suggest one model to be superior to the other, the contrasting results and sensitivity to capital values suggest that using only one statistical model to suggest a preferred annuitisation strategy may result in inappropriate decisions. Hence, consideration of results from multiple models is recommended.

KEYWORDS

Annuities; income drawdown; ruin theory; discounted utility

CONTACT DETAILS

Megan Butler, Alexander Forbes Financial Services, PO Box 787240, Sandton, 2146
E-mail: butlerm@aforbes.co.za
Tel: +27(0)11 269 0710

1. INTRODUCTION

1.1 Background

Levels of choice associated with defined contribution funds generally have been increasing (Mitchell et al., 1999). Globally, members face a number of options as to how to finance their post-retirement spending (Emms, 2010) and increasingly rely on themselves as opposed to financial advisors when making this and other decisions relating to retirement planning (Mitchell et al., 1999). The decision as to how to finance post-retirement spending involves a number of risks including that of pensioners outliving their income (Milevsky & Robinson, 2000; Albrecht & Maurer, 2002), being unable to support a desired consumption level (Levitan, Dolya & Rusconi, unpublished; Emms, 2010) or choosing sub-optimal strategies (Sweeting, 2009).

In South Africa, the risk of inappropriate decumulation leading to old age poverty has been cited by National Treasury as a reason for the proposed reform of the annuities market (National Treasury, 2012). This reform may impact on all retirement fund members. Currently, pension fund members must annuitise at least two-thirds of their accumulated wealth at retirement as opposed to taking benefits as a cash lump sum.[1] National Treasury (2012) has suggested that provident fund members, who are currently not compelled by the Income Tax Act[2] to annuitise, may have to do so in future. In this context, the term 'annuities' includes life annuities as well as income drawdown accounts, commonly known as living annuities in South Africa.

One reform proposal being explored is the introduction of "standardised products into which retirement funds can automatically place members when they retire, without requiring financial advice" (National Treasury, 2012: 13). If the placement is to be made by trustees, the requirement that trustees must act in the "best interest of all members" (Financial Services Board, 2007: 1) would apply. This may imply that trustees would need to identify the optimal placement decision for members and may be jointly and individually liable should a member be able to prove an inappropriate decision was made negligently on the member's behalf.

Currently, the decumulation decision involves a decision around the level of cash lump sum taken at retirement and the life and living annuities purchased at and after retirement. Given the emphasis in the reform proposals on annuitisation as opposed to lump sum cash withdrawals, the decumulation decision may become a choice as to which annuities to purchase and what living annuity strategy to use, which is collectively termed the 'annuitisation' decision for the purposes of this paper.

Statistical and mathematical models, such as ruin theory models (Milevsky & Robinson, 2000; Albrecht & Maurer, 2002; Levitan, Dolya & Rusconi, unpublished) or discounted utility models (Yaari, 1965; Mitchell et al., 1999), can be used to guide the annuitisation decision. Thomson (2003a; 2003b) has proposed the use of discounted utility theory models to aid decision-making in South African defined contribution funds.

1 Income Tax Act (Act 58 of 1962), as amended
2 supra

1.2 Problem Statement and Aim

The aim of the paper is to document the findings of investigations into:

— The highest ranking annuitisation strategies under various circumstances and classes of models;
— Whether different classes of models would suggest different annuitisation decisions for members; and
— The sensitivity of the preferred annuitisation decision to the model parameters.

1.3 Contribution to Knowledge

To the authors' knowledge, the literature on the annuitisation decision in the South African context is limited to a 2009 conference paper by Levitan, Dolya & Rusconi (unpublished). In addition, although there are several international studies on the annuitisation decision, there is a lack of literature on the comparison of the results under different classes of models in the context of annuitisation. Bayraktar & Young (2007) considered income drawdown accounts and the optimal borrowing and lending behaviour of investors under ruin theory and discounted utility metrics and found convergence between models given a known date of death only. However, this result is unhelpful given uncertain life expectancies and, in addition, their discussion ignored life annuities. This paper is expected to contribute to the understanding of the dynamics around the annuitisation decision at a time when reform proposals may require such understanding by actuaries and trustees.

2. LITERATURE REVIEW

2.1 The Risks and Considerations involved in the Annuitisation Decision

The literature suggests that the annuitisation decision involves consideration of a number of different risks as well as consideration of the bequest motive.

Longevity risk, or the risk of outliving available funds before death, has been cited as a key risk by Blake, Cairns & Dowd (2003), Milevsky & Robinson (2000) and Albrecht & Maurer (2002). National Treasury (2012) highlighted this as a key risk of living annuities.

In the context of annuitisation, liquidity risk arises due to the income stream becoming inflexible on the purchase of a life annuity (Milevsky & Robinson, 2000; Albrecht & Maurer, 2002; Sexauer, Peskin & Cassidy, 2012). The elderly require flexible incomes in order to fund sudden and possibly prolonged increases in consumption during retirement arising due to health shocks (Albrecht & Maurer, 2002; Murtaugh, Spillman & Warshawsky, 2001).

Inflation risk, which refers to the risk of expenditure rising faster than income, is considered critical to the annuitisation decision by Mitchell et al. (1999).

The bequest motive is often considered a critical assumption when modelling optimal annuitisation strategies (Yaari, 1965; Albrecht & Maurer, 2002; Sweeting, 2009). However, studies by Hurd (1987) and Shefrin & Thaler (1988) have called into question whether the bequest motive really exists. In addition, Brown (2001) suggested that self-reported bequest motives are not necessarily consistent with the annuitisation strategies that are selected.

Sweeting (2009) cited the importance of the tax and regulatory regimes in influencing the annuitisation decision.

Hence the literature suggests a number of competing considerations are relevant when the individual annuitises. Life annuities hedge longevity risk but introduce liquidity risk and do not explicitly allow for bequests. Inflation risk may be hedged to a greater or lesser degree under different living annuity and life annuity products (Levitan, Dolya & Rusconi, unpublished).

2.2 Discounted Utility and Ruin Theory Approaches to Preferences

The bulk of the literature on the optimisation of annuitisation uses utility maximisation or ruin probabilities, although Sweeting (2009) considered a risk-return trade-off framework. It is, however, noted that quadratic utility is a sufficient condition for analysis in Markowitz-space (Thomson, 2003b) and, hence, Sweeting's analysis is consistent with discounted utility approaches, albeit with a somewhat limiting utility function (Thomson, 2003b). The two dominant approaches are described below.

2.2.1 Discounted Utility

When considering different post-retirement strategies, investors need to make choices regarding the value of different benefits at different points in time. Samuelson (1937) proposed the discounted utility model as a means to examine choices over time, where utility measures the degree of human satisfaction offered by a specific outcome (Fishburne, 1968).

The typical implementation of the model decomposes into the following components:

— An instantaneous utility function which examines how the investor values different quantities of money or goods; and
— A discount factor which accounts for the investor's preference to obtain money or goods sooner rather than later.

Under discounted utility models, the preferred annuitisation strategy is the strategy that maximises discounted utility. However, Samuelson himself stated that it was "completely arbitrary" to assume that individuals would behave so as to maximise discounted utility (Samuelson, 1937: 159). Thomson (2003a), however, asserts that even if discounted utility does not describe actual behaviour, it is valid as a normative theory. In other words, it can be used to describe ideal behaviour given certain axioms.

The six fundamental axioms described in Thomson (2003a) can be described, in the context of this research, as follows:

— Individuals can specify a preference between any two annuitisation options or income streams and bequests.
— Preferences are transitive. In other words, if a first annuitisation option is

preferred to a second and that the second option is preferable to a third, the first option must also be preferable to the third option. This means that rankings are sensible.

— Where outcomes are uncertain, the overall probability of an outcome is important, not the probability of the series of events leading to that outcome.
— If an individual is indifferent between two annuitisation options, they must be indifferent between the outcomes associated with the annuitisation options.
— If offered a choice between two annuitisation options, the annuitant will favour the one with greater probability of favourable outcomes and lower probability of worse outcomes.
— There may be two annuitisation options, one that provides a certain outcome and a second that could provide a number of different options, each with a given probability. If the individual prefers the first option, there must be some fraction that can be applied to scale down the income and bequest stream so that the individual is indifferent between the two options.

There are both practical and theoretical difficulties with discounted utility.

The practical elicitation of utility functions is itself non-trivial. Thomson (2003b) raises the issues of framing bias, which is concerned with how the way the questions are phrased during elicitation can influence the results. In addition, although the elicitation in general involves establishing preferences, Thomson (2003b) suggests there are at least four different theoretical approaches to doing so. Bayraktar & Young (2007) concur that discounted utility models are difficult to parameterise.

Bayraktar & Young (2007) state that ruin theory models may be easier for individuals to understand as the probability of ruin is objective while utility functions are by definition highly subjective.

In terms of theoretical difficulties, Samuelson (1937) commented that the constant discount factor was unrealistic. Frederick, Loewenstein & O'Donoghue (2002) pointed out that constant discount rates and utility functions are problematic and there have been attempts to resolve these difficulties.

Frederick, Loewenstein & O'Donoghue (2002) state that despite these reservations, the discounted utility model was accepted widely and quickly as a descriptively accurate representation of actual behaviour, although Thomson (2003a) indicates that there have been challenges to the descriptive validity of discounted utility theory.

Yaari (1965) and Mitchell et al. (1999) used utility theory to explore the annuitisation decision and Bayraktar & Young (2007) stated that discounted utility is the most common optimisation criterion in the literature.

2.2.2 Ruin Theory

Ruin theory involves the consideration of the probability of entering ruin where this could be defined in various ways. Bayraktar & Young (2007) point out the application of ruin theory both to insurers and other corporate institutions and individuals saving for retirement. Ruin theory might be used to determine optimal strategies for an individual when that individual wishes to avoid running out of funds as their primary objective (Bayraktar & Young, 2007).

Milevsky & Robinson (2000) considered the Lifetime Probability of Ruin ('LPoR') which is the probability of depleting wealth fully before death and Eventual Probability of Ruin ('EPoR') which is the probability of the wealth ever being depleted. Albrecht & Maurer (2002) considered an LPoR measure. Levitan, Dolya & Rusconi (unpublished) considered a slightly modified LPoR by considering the possibility that consumption falls below a threshold level before death.

The preferred strategy under ruin theory is that which minimises the probability of ruin. Bayraktar & Young (2007) have suggested that minimising the probability of running out of funds is intuitively more appealing than maximising utility due to the fact that a probability is more objective than a value calculated using a subjective utility function. However, it is worth noting that ruin theory requires stochastic simulations which may involve subjectively parameterised models, and hence ruin theory is not strictly objective.

2.3 Preferred Strategies

2.3.1 Discounted Utility

Discounted utility models found in the literature suggest a range of results depending on the level of risk aversion of the individual and the bequest motive.

Yaari (1965) explored how a rational retiree would seek to maximise utility given a starting level of wealth and the constraint that the asset value at death must be non-negative. Yaari (1965) found that in the absence of a bequest motive, the rational strategy would be to annuitise fully, as opposed to investing in an income drawdown account. Blake, Cairns & Dowd (2003) were able to explore the annuitisation problem more fully given advances in computing and new investment products.

Blake, Cairns & Dowd (2003) considered three annuitisation options:

— A level life annuity;
— An equity-linked annuity with a level life annuity purchased thereafter. The equity-linked life annuity provides an investment-linked income together with mortality credits to hedge against mortality risks; and
— An equity-linked drawdown account with a level life annuity purchased thereafter. Unlike the equity-linked annuity, the equity-linked drawdown account does not provide mortality credits.

Blake, Cairns & Dowd (2003) found that life annuities are preferable for risk-averse pensioners and that income drawdown accounts are more suitable for risk-seeking pensioners. However, the choice of equity exposure in the equity-linked drawdown account and equity-linked annuity may be an even more important decision than the choice between the annuity and the drawdown account.

2.3.2 Ruin Theory

Much of the literature on ruin theory models in the context of annuitisation considers the optimal asset allocation in the income drawdown account as opposed to the balance between life annuities and income drawdown accounts. Milevsky & Robinson (2000) and Albrecht & Maurer (2002) considered the asset allocation problem in the Canadian and German contexts respectively.

Milevsky & Robinson (2000) and Albrecht & Maurer (2002) both considered the complete exhaustion of funds in an income drawdown account. Albrecht & Maurer (2002) used drawdown rates set with reference to income that could be earned under a with-profit life annuity while Milevsky & Robinson (2000) use an arbitrary drawdown rate. Milevsky & Robinson (2000) found that females had much higher probabilities of ruin than males and each sex had a different optimal investment strategy although all investors benefitted from diversification. Albrecht & Maurer (2002) similarly found that the ruin probability was minimised by holding a diversified portfolio. Higher post-retirement interest rates, and hence lower initial drawdown rates, were associated with lower exposure to growth assets (Albrecht & Maurer, 2002).

Levitan, Dolya & Rusconi (unpublished) used ruin theory to explore the trade-offs between life annuities and income drawdown accounts with various investment strategies. Ruin was defined in terms of income falling below the level required to sustain a desired level of spending and hence the drawdown rates were set according to this expenditure level (Levitan Dolya & Rusconi, unpublished). The annuitisation decision involved consideration of four strategies:

- A life annuity level in nominal terms;
- A life annuity increasing at 3% p.a.;
- An inflation-linked life annuity; and
- An income drawdown facility. Four levels of equity exposure in the income drawdown account were tested, namely 0%, 25%, 50% and 75%, with the balance of the assets invested in conventional fixed-interest stock.

Levitan, Dolya & Rusconi (unpublished) found that the results were very sensitive to the ratio of accumulated credit to annual income requirement.

2.4 Sensitivity of Annuitisation Preferences to Various Demographic Parameters

The literature on the annuitisation decision suggested that various annuitisation options may become more or less desirable depending on individual circumstances. The income preferences relative to initial wealth, bequest motive, utility function adopted, mortality and other wealth are discussed in turn.

2.4.1 Income Preferences relative to Initial Wealth

Albrecht & Maurer (2002), Blake, Cairns & Dowd (2003) and Emms (2010) found that higher income preferences lend themselves to higher equity exposure, given that an income drawdown account is purchased at retirement.

Levitan, Dolya & Rusconi (unpublished) found that where the accumulated wealth was sufficient to secure an inflation-linked annuity, this was the most preferable strategy. However, if this was unaffordable, the income drawdown accounts provided lower ruin probabilities than the life annuities. The higher the income requirement relative to the capital at retirement, the higher the equity component required in the income drawdown account, which was consistent with Albrecht & Maurer (2002), Blake, Cairns & Dowd (2003) and Emms (2010).

2.4.2 The Bequest Motive

Although low purchase rates of life annuities are often attributed to the bequest motive (Davidoff, Brown & Diamond, 2005), Yaari (1965) suggested that if annuities are available individuals can separate the bequest and consumption motives. In other words, the bequest motive can be accommodated via a cash withdrawal at retirement as opposed to influencing the annuitisation decision. Davidoff, Brown & Diamond (2005) similarly established that most mathematical models of the bequest motive fail to explain low rates of life annuity purchases.

An important result from Blake, Cairns & Dowd (2003) was that the choice of annuity product was not sensitive to the bequest motive. However, the strength of the bequest motive may influence how much equity should be held post retirement and the age at which the individual eventually purchases a life annuity. Greater bequest motives suggest a later age at which a life annuity is purchased after investment in an income drawdown account as well as higher equity exposure (Blake, Cairns & Dowd, 2003). The latter was also concluded by Emms (2010).

2.4.3 Utility Function and Relative Risk Aversion

Blake, Cairns & Dowd (2003) suggested that whether an exponential or power utility function was used was less important than the value of the relative risk aversion parameter. They tested the annuitisation preferences under a range of relative risk aversion ('RRA') parameters. Unsurprisingly, risk-seekers with low RRA parameters were found to prefer 100% equity income drawdown accounts. For an investor with moderate RRA, income drawdown accounts were still preferable to annuities but the equity exposure decreased with increasing risk aversion. For high RRA parameters, which represented the very risk-averse, life annuities were found to be preferable.

These results are consistent with Sweeting (2009) who considered risk-return trade-offs, where the return criterion was the pension in excess of what could be earned on a decumulation strategy of a compulsory-purchase fixed annuity and the risk criterion was the Value at Risk. Sweeting (2009) found that for relatively low levels of risk aversion an income drawdown account converting to a life annuity later in retirement was preferable.

2.4.4 Mortality
Blake, Cairns & Dowd (2003), Albrecht & Maurer (2002) and Levitan, Dolya & Rusconi (unpublished) all considered only male lives. However, results from Milevsky & Robinson (2000) suggest that female lives, with lighter mortality, should have higher equity allocations than shorter-life males. Blake, Cairns & Dowd (2003) tested their results for impaired lives and found, unsurprisingly, that impaired lives may benefit from strategies that accelerate the payment of their benefits.

2.4.5 Other Wealth
Blake, Cairns & Dowd (2003) also found the level of non-retirement funding wealth made little difference as to the preferred annuitisation strategy as did the introduction of a fixed state pension. Although the former result may seem surprising, non-retirement assets may consist largely of illiquid assets such as property which would not influence income levels.

3. MODELS
Whilst the discounted utility approach has well-documented shortcomings, it is mathematically tractable, has been used widely and arguably has an intuitive appeal. On this basis the authors believe it offers an interesting basis of comparison against the ruin theory approach, which is perhaps considered more objective and has been used widely in recent literature on annuitisation. A number of annuitisation strategies and scenarios were considered; these are set out in Sections 3.1 and 3.2 respectively. Section 3.3 sets out how the income levels under each strategy were evaluated. The discounted utility and ruin theory models are set out in Sections 3.4 and 3.5 respectively.

3.1 Annuitisation Strategies
The attractiveness of different annuitisation strategies was assessed under the discounted utility and ruin theory frameworks introduced earlier. Eleven strategies were considered and are summarised in Table 3.1. The list of strategies is not exhaustive and can be expanded in future research.

For simplicity the risk of the insurer defaulting is ignored.

The life annuities all included a 75% spouse's reversion which would result in any surviving spouse receiving 75% of the prevailing income when the principal pensioner dies. This would allow the widowed spouse to meet their reduced variable costs as well as bear the fixed costs associated with running a household that do not reduce when the principal pensioner passes away. The life annuities also included 10-year guarantee period so annuity payments continue for at least 10 years after purchase. Anecdotally, this is consistent with retail annuity purchase behaviour.

For simplicity, no asset classes were considered beyond local equity and fixed-interest which represent risky and risk-free asset classes respectively. The allocations between equity and fixed-interest were the same as was adopted by Milevsky & Robinson (2000), Blake, Cairns & Dowd (2003), Levitan, Dolya & Rusconi (unpublished).

Table 3.1 Annuitisation strategies considered

Strategy description	Abbreviation
Purchase a level annuity at retirement, where payments remain constant.	Level
Purchase a fixed-escalation annuity, where payments increase by 5% each year. The 5% figure was chosen to be slightly above the middle of the 3%–6% inflation target band used by the South African Reserve Bank	Fixed 5%
Purchase an inflation-linked annuity at retirement, where payments increase by the inflation rate each year.	IL
A living annuity strategy annuitising into an inflation-linked annuity at age 75. Prior to annuitisation, the portfolio has a 100% allocation to fixed-interest instruments.	LwA 0/100
A living annuity strategy annuitising into an inflation-linked annuity at age 75. Prior to annuitisation, the portfolio has a 25% allocation to equities and a 75% allocation to fixed-interest instruments.	LwA 25/75
A living annuity strategy annuitising into an inflation-linked annuity at age 75. Prior to annuitisation, the portfolio has a 50% allocation to equities and a 50% allocation to fixed-interest instruments.	LwA 50/50
A living annuity strategy annuitising into an inflation-linked annuity at age 75. Prior to annuitisation, the portfolio has a 75% allocation to equities and a 25% allocation to fixed-interest instruments.	LwA 75/25
A living annuity strategy without annuitisation, the portfolio has a 100% allocation to fixed-interest instruments.	L 0/100
A living annuity strategy without annuitisation, the portfolio has a 25% allocation to equities and a 75% allocation to fixed-interest instruments.	L 25/75
A living annuity strategy without annuitisation, the portfolio has a 50% allocation to equities, 50% allocation to fixed-interest instruments.	L 50/50
A living annuity strategy without annuitisation, the portfolio has a 75% allocation to equities, 25% allocation to fixed-interest instruments.	L 75/25

UK regulation enforces the purchase of a life annuity at age 75 (Sweeting, 2009). In order to be consistent with Blake, Cairns & Dowd (2003), the age of 75 was used as the age at which the life annuity was purchased in the living annuity with annuitisation, which is referred to as 'the lifestage annuity' in this research. However, it was noted that Blake, Cairns & Dowd (2003) suggest that the optimal age to purchase a life annuity is very sensitive to risk aversion levels, slightly sensitive to the bequest motive and depends on the size of the fund at the annuitisation point. Further investigation of the optimal age to purchase a life annuity is left for future research.

3.2 Scenarios

A range of demographic profiles were selected, which were believed to be reasonably realistic in the South African context. The purpose was to assess whether the optimal annuity choice would be influenced significantly by changes in the demographic profile. The scenarios were selected to be reasonable and to produce a range of ruin probabilities. The exact choice is admittedly arbitrary, and for this reason a number of scenarios were run.

Table 3.2 Member scenarios

Case	Main member age	Main member gender	Spouse age	Income requirement in first year as a percentage of initial accumulated wealth	
				Necessities case	Comfort case
1	65	Male	61	5.16%	6.60%
2	60	Male	56	5.16%	6.60%
3	65	Male	–	5.16%	6.60%
4	65	Female	69	5.16%	6.60%
5	65	Male	61	4.20%	5.64%
6	65	Male	61	6.00%	7.44%

The comfort case refers to income levels sufficient to allow the individual to live without noticeable economies, which are required in the necessities case (Levitan, Dolya & Rusconi, unpublished). The base case retirement age of 65 was both consistent with Blake, Cairns & Dowd (2003) and the modal retirement age observed from administrator data.[3]

The income requirement was approximately 13,4–19,4 times the annual income requirement, which is slightly higher than the value of 14 used in Milevsky & Robinson (2000).

3.3 Generating the Income Streams for each Annuitisation Strategy

For the life annuity strategies, Level, Fixed 5% and IL, the initial income levels were determined using actual quotes in the market, valid from 1 July–7 July 2012, published on 29 June 2012. The Fixed 5% income stream was increased at a rate of 5% pa and the IL income stream was increased by a stochastically generated inflation parameter.

For living annuities, the drawdown during the member's lifetime is managed according to a rules-based system. The member chooses at the outset whether to draw down at the rate of income required for either comfort or necessities. This drawdown

3 Alexander Forbes Member Watch 2011, July 2012

is limited by a minimum of 2,5% of fund credit per annum and a maximum of 17,5% of fund credit per annum as per the Income Tax Act. Although a drawdown rate that is a fixed proportion of the living annuity fund, as opposed to a proportion selected to meet an income need may delay ruin (Emms, 2010), it can be argued that the budgetary needs of households are in currency terms and not strictly determined by the size of the living annuity.

The drawdown account at the start of the year is immediately reduced by the drawdown and increased by a stochastically simulated return derived using the Maitland Stochastic Investment Model (Maitland, 2010), based on the asset allocations for the strategy.

For the purposes of this research, the Maitland Stochastic Investment Model was parameterised based on bond yields as at 31 March 2012. The model was calibrated to a 10-year yield on nominal bonds of 8,3% per annum and a nominal 10-year expected return on equities of 11,3% per annum. This was based on a long-term equity risk premium of 3%, which is a rounded-off value from Hu (unpublished). The model was calibrated to give an expected inflation rate of 5,8% per annum on average over ten years, based on the difference between 10-year nominal and inflation-linked bond yields and a 0,5% inflation-risk premium on nominal bonds. Hu (unpublished) had estimated the inflation-risk premium in South Africa at 1,3% in the early 2000s, but this was done at the launch of inflation-linked bonds as an asset class in South Africa, when the inflation-targeting regime was not as well established. Hu (unpublished) warned that this was a crude estimate, which was high relative to international research and may have reflected market uncertainty at the time on whether inflation would be kept under control. Hence, the authors believe a 0,5% inflation risk premium to be justifiable given the greater maturity of the index inflation-linked bond market, and the entrenched policy of inflation-targeting by the South African Reserve Bank.

2 500 simulations were completed for each strategy involving a living annuity or lifestage annuity.

For the lifestage annuity, the annuity price at the date of purchase was estimated using the stochastically simulated index-linked bond yields prevailing at that time.

The lifetime of the pensioner is stochastically simulated based on a mortality table of PA(90) rated down by 3 years for males and 2 years for females and with a 1.5% p.a. mortality improvement from 2012.

3.4 The Discounted Utility Models

3.4.1 Model Formulation

The discounted utility model is based on the framework set out by Blake, Cairns & Dowd (2003), with some modifications for South African conditions. In general, the discounted utility framework involves identifying the strategy with the highest discounted utility, U^*, such that

$$U^* = \int_0^b U(x) e^{-\beta t} dt \tag{3.1}$$

where

b	is the end of the time period considered
x	is income
$U(x)$	is the utility function
β	is the force of discount
t	is time

For the purposes of this investigation, a modified power function was used for the utility function. The value function, or discounted utility function, denoted *V(s,f)* is given by the expression in Equation 3.2.

$$V(s,f) = E\left[\sum_{t=s}^{K} e^{-\beta t} J_1\left(P(t)\right) + k_2 e^{-\beta(K+1)} J_2\left(D(K+1)\right) \mid F(s) = f, \text{alive at } s\right] \tag{3.2}$$

where

$F(s)$	is the accumulated pension wealth *s* years after retirement
s	is the number of years since retirement
β	is the force of discount
K	is the curtate future lifetime of the member at the date of retirement
$J_1(P(t))$	is the utility of consumption
$P(t)$	is the pension in year *t*
k_2	is the preference for bequests
$J_2(D(t))$	is the utility of bequests
$D(t)$	is the bequest payable at time *t* given that death occurred within the year ending at time *t*

A constant relative risk aversion model was used for the utility of consumption, $J_1(P(t))$ as given in Equation 3.3.

$$J_1\left(P(t)\right) = h_1\left(\gamma_1\right)\left(\frac{P(t)}{P_B}\right)^{\gamma_1} \tag{3.3}$$

where

P_B	is the base pension
$1-\gamma_1$	is the relative risk aversion associated with consumption

and where $h_1(\gamma_1)$ is given by Equation 3.4

$$h_1\left(\gamma_1\right) = \frac{1}{1 - d_1^{\gamma_1}} \tag{3.4}$$

where

d_1 is a shape parameter for the consumption utility curve

For the utility of bequests, $J_2(D(t))$, a function from the hyperbolic absolute risk aversion class was used, given by Equation 3.5. Blake, Cairns & Dowd (2003), Yaari (1965) and Levitan, Dolya & Rusconi (unpublished) do not deal with reversionary annuities. For the purposes of this research, annuity income to a spouse was treated as a series of bequests. Further research is required to test whether this is reasonable.

$$J_2\left(D(t)\right) = h_2\left(\gamma_2\right)\left[\left(\frac{D(t)+d_2}{d_2}\right)^{\gamma_2} - 1\right], \text{ with } d_2 > 0 \tag{3.5}$$

where

$1-\gamma_2$ is the relative risk aversion associated with bequests

d_2 is the value of assets held outside the pension fund such as a house

and where $h_2(\gamma_2)$ is given by Equation 3.6

$$h_2(\gamma_2) = \frac{1}{\left(\frac{F(0)+d_2}{d_2}\right)^{\gamma_2} - 1} \tag{3.6}$$

3.4.2 Calculation of Discounted Utility

For the strategies considered, the income receivable each year is stochastically simulated using transformations of the Maitland Stochastic Investment Model. Based on the simulated income and the bequest or bequests made, the member's discounted utility is calculated. The average utility over the 2 500 simulations is then used as an estimate for the expected utility for the strategy under consideration.

3.4.3 Parameterisation

When parameterising the model, care was taken to be consistent with the literature, particularly with Blake, Cairns & Dowd (2003), in order to allow some comparability of results. The specific parameters adopted are described as follows.

3.4.3.1 Risk aversion parameters

The relative risk aversion parameter is represented by $1-\gamma_1$. A higher value represents a more risk-averse person. Blake, Cairns & Dowd (2003) adopted a range from 0,25 to 25 in order to accommodate both very risk-averse and very risk-tolerant preferences and adopted a base case scenario value of 3.96. For the purposes of this research the same base case value was used. To test the sensitivity of this parameter, six point estimates were taken from this range, namely 0,3, 5,2, 10,2, 15,1, 20,1 and 25. These were chosen to be equally distant from one another by a factor of 4,95. As per Blake, Cairns & Dowd (2003), γ_1 was set equal to γ_2.

3.4.3.2 Discount rate
A flat real yield of 2,45% was used which represented the average of the yields of the two longest-dated SA government-issued inflation-linked bonds in the market.[4] A real yield was chosen given the need to protect income against inflation risk. Although, more recently, real yields have declined, it is shown in 4.2.2 that the results would not be materially affected. In the modelling this rate of interest was converted into a force of interest of 2,44%, for application in the model. For simplicity, this real yield was used as an intertemporal discount rate. Blake, Cairns & Dowd (2003) used a nominal intertemporal discount rate chosen to be consistent to the rate used by the government actuary to value benefits. Sensitivity testing was also performed using forces of interest of 0,1% and 10%.

3.4.3.3 Initial fund credit
The average and median retirement benefits for Alexander Forbes clients were approximately R1 130 000 and R480 000 in 2011.[5] A figure of R1 million was used at the outset for *F(0)*. This resulted in income levels that fell below the income tax thresholds. This limits the application of the results to higher income earners as Sweeting (2009) has shown that tax regimes can influence the annuitisation decision.

3.4.3.4 Bequests and other assets
The base case value for the bequest motive parameter, k_2, was 5. In sensitivity testing values of 1 and 10 were tested. For the other assets, a value of R1 million was used for simplicity and values of R500 000 and R2 million were adopted for sensitivity testing.

3.4.3.5 Base pension
$J_1(P(t))$ includes a ratio of income to that derived from a base pension. Three base pensions were used for the parameter P_B. As per Blake, Cairns & Dowd (2003), a level life annuity was adopted for the first case and the resultant discounted utility model is referred to as 'DUL' for the purposes of this paper. In the second, an inflation-linked life annuity was adopted and the resultant model is termed 'DUI.' In the third, the necessity case spending level, increased by inflation, and termed 'DUN', was used as per Levitan, Dolya & Rusconi (unpublished).

3.4.3.6 Shape parameter, d_1
A value of 0.75 was chosen as per Blake, Cairns & Dowd (2003), who remarked that this parameter value could be chosen freely in the range of between 0 and 1 to adjust the shape of the utility curve. To test for sensitivity, values of 0,05 and 0,95 were also used.

4 Bloomberg, June 2012
5 Alexander Forbes, supra

3.5 The Ruin Theory Model

The ruin theory approach involved finding the strategy with the lowest LPoR where ruin is defined as income falling below a level of income for comfort, or for necessity, increased by simulated inflation each year, as per Levitan, Dolya & Rusconi (unpublished). If the income available from the annuitisation strategy is lower than the income needed, while the pensioner is alive, the strategy is deemed to have resulted in ruin. The count of the simulations resulting in ruin was divided by 2 500 to give the probability of ruin.

4. RESULTS

Section 4.1 contains an analysis of the results using the standard basis. Section 4.2 provides the results of the sensitivity tests.

4.1 Results on the Standard Basis

4.1.1 Scenario 1: Base Scenario

The results for the base case of a man aged 65 with a wife aged 61 are given in Table 4.1.

Table 4.1 Results for the base case scenario

	Comfort income: R5 500 pm					Necessity income: R4 300 pm				
	Ruin Theory		Discounted Utility			Ruin Theory		Discounted Utility		
	% Ruin	Ranking	DUL	DUI	DUN	% Ruin	Ranking	DUL	DUI	DUN
Level	83%	9	1	3	8	69%	10	1	5	10
Fixed 5%	100%	10	4	1	6	20%	1	2	1	8
IL	100%	10	6	2	7	100%	11	6	4	9
LwA 0/100	74%	8	3	5	4	51%	9	4	3	7
LwA 25/75	71%	7	2	4	3	33%	7	3	2	5
LwA 50/50	60%	5	5	6	2	30%	3	5	6	3
LwA 75/25	53%	3	8	7	1	32%	6	9	7	2
L 0/100	69%	6	9	10	10	50%	8	8	9	6
L 25/75	55%	4	7	8	5	30%	3	7	8	1
L 50/50	51%	2	10	9	9	28%	2	10	10	4
L 75/25	48%	1	11	11	11	30%	3	11	11	11

When the required level of income is that to meet comfort, ruin theory favours aggressive living annuity strategies with a 75% allocation to equities, resulting in a probability of ruin of 48%. Other aggressively managed strategies also fared well, as investing aggressively increases the probability of achieving the desired income for comfort from living annuities. By not annuitising and not being locked into fixed interest instruments, returns were expected to be higher in pure living annuity strategies, albeit by running the risk of running out of income, possibly by a significant margin.

In contrast, locking into income from a fixed 5% escalation or inflation-linked annuity was certain to result in ruin, because the incomes from these annuities were lower than the level of desired income for comfort. Importantly, the ruin theory criterion does not take the extent of shortfall into account and hence treats a shortfall of R1 and shortfall of R10 000 equally.

When the required level of income is that for necessities only, the highest ranked strategy under ruin theory now switches to the fixed 5% escalation annuity, as the income purchasable from the life annuity in general exceeds the level of income for necessity. The inflation risk remains which accounts for the 20% probability of ruin. This strategy is followed by a fairly aggressively-managed living annuity, albeit less aggressive than when the income threshold for comfort was used. Again, an inflation-linked strategy remained the lowest ranked as income from it is still less than the level for necessity.

The discounted utility results are strongly influenced by the choice of the base pension. This is due to the formula used to define the utility function, and in particular the expression given in Equation 3.3, which involves the ratio of income to the base pension being raised to an exponent. Hence the overall utility is very sensitive to the relative size of any shortfall to the base pension. By taking utility into account, strategies providing higher levels of income upfront are favoured. In sensitivity testing of the discount rate parameter, summarised in 4.2.2, it was found that this impact on the rankings of varying base pensions was much greater than varying different discount rates.

Under DUL, level annuities emerged as most preferred. There was not much difference in the interpretations between the income for comfort and necessities cases, under DUL.

It is noteworthy that the fixed-escalation at 5% annuity emerged as the most preferred strategy under DUI, for both income for comfort and necessities. This suggests that the slightly higher income from the fixed 5% escalation relative to inflation-linked annuity made it relatively more appealing, even though ongoing increases may not match inflation, which in 48,0% of simulations was above 5% p.a. and was in one case as high as 20,9% p.a. Similar to the DUL result, the purchase of life annuities is favoured, and more conservative asset allocations were preferred to more aggressive ones.

The picture was somewhat different under DUN, which appeared to be the most risk-seeking of the discounted utility measures. When income for comfort is used, the most preferred strategy was the high-equity living annuity with an inflation-linked annuity purchased at age 75. When income for necessities is used, this moves to living annuity without annuitisation with more conservative asset allocations. The reason is because when the threshold is income for necessities, which is lower than for comfort, less is drawn each year from the living annuity, which means less capital is depleted, making the living annuity arrangement more sustainable. Hence, annuitisation is not as valued.

4.1.2 Scenario 2: Lower Retirement Age Scenario

The ruin probabilities and strategy rankings for the case of a man aged 60 with a spouse aged 56 are given in Table 4.2.

Table 4.2 Results for the retirement age 60 scenario

	Comfort income: R5 500 pm					Necessity income: R4 300 pm				
	Ruin Theory		Discounted Utility			Ruin Theory		Discounted Utility		
	% Ruin	Ranking	DUL	DUI	DUN	% Ruin	Ranking	DUL	DUI	DUN
Level	91%	10	1	3	7	81%	10	1	4	10
Fixed 5%	100%	7	2	1	5	55%	7	2	1	7
IL	100%	11	5	2	6	100%	11	7	2	8
LwA 0/100	82%	9	4	5	3	71%	9	4	5	5
LwA 25/75	75%	6	3	4	2	51%	6	3	3	3
LwA 50/50	67%	4	6	6	1	39%	4	5	6	1
LwA 75/25	60%	2	9	7	4	38%	2	10	7	4
L 0/100	82%	8	8	9	9	65%	8	8	9	6
L 25/75	71%	5	7	8	8	48%	5	6	8	2
L 50/50	62%	2	10	10	10	38%	2	9	10	9
L 75/25	56%	1	11	11	11	35%	1	11	11	11

For an earlier retirement age, the ruin theory results were mostly unchanged except the fixed escalation annuity was not as highly ranked when considering the necessity income case. This is due to the higher cost of the annuity at younger ages.

Under the DUL, level annuities emerged as the most preferred once again for both income requirement levels. Similarly, the favoured strategy under DUI remained fixed escalation life annuities. Under DUN, the more aggressively-managed lifestage annuities were generally favoured.

4.1.3 Scenario 3: Single Member Scenario

The ruin probabilities and strategy rankings for the case of a man aged 65 is given in Table 4.3.

Under ruin theory, for a single male aged 65, inflation-linked annuities were the best strategies for income for comfort or necessity, as both incomes were affordable under the annuity which no longer included a spouse’s reversion. The least preferred strategy was for level annuities, due to the effects of inflation eroding the income purchased from the annuity that was initially higher than the income thresholds.

Under the DUL, level annuities emerged as the most preferred once again for both income requirement levels. Similarly, the favoured strategy under DUI remained fixed escalation life annuities. Under DUN, low equity living annuities without annuitisation were favoured.

Table 4.3 Results for the single male scenario

	Comfort income: R5 500 pm					Necessity income: R4 300 pm				
	Ruin Theory		Discounted Utility			Ruin Theory		Discounted Utility		
	% Ruin	Ranking	DUL	DUI	DUN	% Ruin	Ranking	DUL	DUI	DUN
Level	78%	11	1	5	8	63%	11	1	5	10
Fixed 5%	27%	2	2	1	6	2%	2	2	1	8
IL	0%	1	5	2	7	0%	1	3	2	9
LwA 0/100	72%	10	4	4	4	10%	4	5	4	5
LwA 25/75	51%	6	3	3	2	8%	3	4	3	3
LwA 50/50	42%	3	6	6	3	13%	5	6	6	4
LwA 75/25	42%	3	7	7	5	19%	6	8	7	6
L 0/100	70%	9	9	9	9	49%	10	9	9	2
L 25/75	60%	8	8	8	1	35%	9	7	8	1
L 50/50	51%	6	10	10	10	29%	7	10	10	7
L 75/25	48%	5	11	11	11	30%	8	11	11	11

4.1.4 Scenario 4: Female Main Member Scenario

When the analysis was changed to a female pensioner with an older husband, the relative rankings were similar to the base case, as shown in Table 4.4.

The only noteworthy difference from the base case was for the DUN model which favoured a lifestage annuity but with a 50% equity exposure under the comfort income case.

4.1.5 Scenario 5: Lower Income Requirement Scenario

The analysis then returned to the base profile of male aged 65 and female 61, but with the income levels lowered. The results are shown in Table 4.5.

Under ruin theory, the absolute probabilities of ruin dropped, but the relative rankings amongst the strategies remained similar relative to the base case. With lower income requirements, conservative living annuities without annuitisation were preferred under DUN, however the fixed 5% escalation annuity was still the highest ranked strategy under DUI.

TABLE 4.4 Results for female member scenario

	Comfort income: R5 500 pm					Necessity income: R4 300 pm				
	Ruin Theory		Discounted Utility			Ruin Theory		Discounted Utility		
	% Ruin	Ranking	DUL	DUI	DUN	% Ruin	Ranking	DUL	DUI	DUN
Level	84%	9	1	4	8	71%	11	1	5	10
Fixed 5%	100%	10	4	1	6	15%	2	2	1	8
IL	100%	10	6	2	7	0%	1	3	3	9
LwA 0/100	76%	8	3	5	4	40%	9	5	4	6
LwA 25/75	69%	6	2	3	3	25%	3	4	2	5
LwA 50/50	56%	4	5	6	1	26%	4	6	6	4
LwA 75/25	51%	2	8	7	2	29%	5	9	7	3
L 0/100	70%	7	9	9	9	50%	10	8	9	2
L 25/75	62%	5	7	8	5	36%	8	7	8	1
L 50/50	51%	2	10	10	10	29%	5	10	10	7
L 75/25	48%	1	11	11	11	29%	5	11	11	11

TABLE 4.5 Results for the lower income scenario

	Comfort income: R4 700 pm					Necessity income: R3 500 pm				
	Ruin Theory		Discounted Utility			Ruin Theory		Discounted Utility		
	% Ruin	Ranking	DUL	DUI	DUN	% Ruin	Ranking	DUL	DUI	DUN
Level	75%	10	1	5	10	57%	11	1	3	10
Fixed 5%	47%	6	2	1	8	3%	2	2	1	8
IL	100%	11	6	3	9	0%	1	3	2	9
LwA 0/100	71%	9	4	4	6	11%	4	5	5	7
LwA 25/75	54%	7	3	2	4	8%	3	4	4	6
LwA 50/50	44%	4	5	6	3	12%	5	6	6	5
LwA 75/25	42%	3	9	7	2	17%	8	9	7	4
L 0/100	57%	8	8	9	7	29%	10	8	9	3
L 25/75	45%	5	7	8	1	15%	7	7	8	1
L 50/50	36%	1	10	10	5	13%	6	10	10	2
L 75/25	36%	1	11	11	11	18%	9	11	11	11

4.1.6 *Scenario 6: Higher Income Requirement Scenario*
The results for the couple with higher income requirements are shown in Table 4.6.

Table 4.6 Results for the higher income scenario

	Comfort income: R6 200 pm					Necessity income: R5 000 pm				
	Ruin Theory		Discounted Utility			Ruin Theory		Discounted Utility		
	% Ruin	Ranking	DUL	DUI	DUN	% Ruin	Ranking	DUL	DUI	DUN
Level	89%	9	1	3	7	78%	9	1	5	10
Fixed 5%	100%	10	3	1	5	79%	10	3	1	6
IL	100%	10	6	2	6	100%	11	6	4	7
LwA 0/100	78%	8	4	5	4	73%	8	4	3	4
LwA 25/75	76%	6	2	4	3	58%	6	2	2	3
LwA 50/50	70%	4	5	6	2	48%	4	5	6	2
LwA 75/25	64%	3	7	7	1	44%	3	8	7	1
L 0/100	76%	6	10	10	10	62%	7	9	9	9
L 25/75	70%	4	8	8	8	50%	5	7	8	5
L 50/50	62%	2	9	9	9	41%	1	10	10	8
L 75/25	59%	1	11	11	11	41%	1	11	11	11

Under ruin theory, when the income requirements were raised, the absolute probabilities of ruin increased, but the relative rankings again remained similar to the base case. The DUI and DUL rankings remain unchanged but the DUN favoured annuitisation and more equity exposure when the living annuities were set to deliver the necessity level of income.

4.2 Sensitivity Tests of DUL Results
The sensitivity of the results from the discounted utility models has already been tested for variations in the base pension and differences in demographic profiles. The sensitivities of five other key parameters in the DUL model were also tested, namely:
- RRA, $1-\gamma_1$
- Force of discount for consumption, β
- Shape parameter for h_1 and J_1 the consumption utility, d_1
- Assets outside retirement fund, d_2
- Bequest motive, k_2

For presentation purposes, the relative rankings of the 11 strategies under these sensitivity tests are shown.

4.2.1 *Sensitivity Test on Relative Risk Aversion*

The rankings for various levels of the relative risk aversion parameters are shown in Table 4.7.

Table 4.7 Rankings for various RRA parameters

	Comfort income						Necessity income					
RRA	0,3	5,2	10,2	15,1	20,1	25	0,3	5,2	10,2	15,1	20,1	25
Level	2	1	1	1	1	1	2	1	1	1	1	1
Fixed 5%	8	4	2	2	2	2	4	2	2	2	2	2
IL	11	5	4	4	3	3	10	5	5	4	4	4
LwA 0/100	9	2	3	3	4	4	8	4	3	3	3	3
LwA 25/75	7	3	5	5	5	5	5	3	4	5	5	5
LwA 50/50	4	6	6	6	6	6	3	6	6	6	6	6
LwA 75/25	1	8	7	7	7	7	1	9	9	9	8	8
L 0/100	10	9	9	9	9	9	11	8	7	7	7	7
L 25/75	6	7	8	8	8	8	9	7	8	8	9	9
L 50/50	5	10	10	10	10	10	7	10	10	10	10	10
L 75/25	3	11	11	11	11	11	6	11	11	11	11	11

For highly risk-seeking annuitants with RRA parameters of 0,3, the aggressive living annuity with annuitisation was preferred. However, once more risk aversion is introduced, above 5,2 there is no change in the rankings of the attractiveness of these strategies.

4.2.2 *Sensitivity Test on Force of Discount*

The rankings under various discount rates are shown in Table 4.8.

It could be seen that under both the income for comfort and necessities scenarios, even very large differences in the force of discount has little effect on the rankings across the strategies. This is attributable to the sensitivity of ratio of annuitisation income to the base pension level being high. Relatively speaking, the impact of the discount rate is much lower. Hence, although the absolute utility levels changed with a change in the discount rate, the rankings did not change much.

4.2.3 *Sensitivity Test on Shape Parameter*

The rankings under various shape parameters are shown in Table 4.9

The range recommended by Blake, Cairns & Dowd (2003) lay between 0 and 1. With the base at 0,75, higher values had virtually no effect on the rankings. However, for lower values, there was a significant effect, with those values favouring more aggressive living annuity strategies as opposed to level annuities and more conservative strategies.

Table 4.8 Results for various forces of discount

	Comfort income			Necessity income		
B	0,10%	2,44%	10%	0,10%	2,44%	10%
Level	1	1	1	1	1	1
Fixed 5%	3	4	6	2	2	2
IL	6	6	8	6	6	8
LwA 0/100	4	3	3	4	4	4
LwA 25/75	2	2	2	3	3	3
LwA 50/50	5	5	5	5	5	7
LwA 75/25	7	8	10	9	9	10
L 0/100	9	9	7	8	8	6
L 25/75	8	7	4	7	7	5
L 50/50	10	10	9	10	10	9
L 75/25	11	11	11	11	11	11

Table 4.9 Results for various shape parameters

	Comfort income			Necessity income		
d_1	0,05	0,75	0,95	0,05	0,75	0,95
Level	11	1	1	11	1	1
Fixed 5%	9	4	4	9	2	2
IL	10	6	6	10	6	5
LwA 0/100	8	3	3	8	4	4
LwA 25/75	7	2	2	7	3	3
LwA 50/50	5	5	5	6	5	6
LwA 75/25	4	8	8	5	9	9
L 0/100	6	9	9	4	8	8
L 25/75	3	7	7	2	7	7
L 50/50	1	10	10	1	10	10
L 75/25	2	11	11	3	11	11

4.2.4 Sensitivity Test on Assets Outside Fund

Other values tested were R500 000 and R2 million, in addition to the base of R1 million. There was no effect on the rankings and no material changes in the utility levels and hence the tabulated results are not shown.

4.2.5 Sensitivity Test on Bequest Motive

Other values tested were 1 and 10, in addition to the base of 5. The results are shown in Table 4.10.

Table 4.10 Results for various strengths of bequest motive

	Comfort income			Necessity income		
k_2	1	5	10	1	5	10
Level	1	1	1	1	1	1
Fixed 5%	4	4	4	2	2	2
IL	6	6	6	5	6	7
LwA 0/100	3	3	3	4	4	4
LwA 25/75	2	2	2	3	3	3
LwA 50/50	5	5	5	6	5	5
LwA 75/25	8	8	8	9	9	9
L 0/100	9	9	9	8	8	8
L 25/75	7	7	7	7	7	6
L 50/50	10	10	10	10	10	10
L 75/25	11	11	11	11	11	11

Interestingly, varying the strength of bequest motive had only a slight impact under the income for necessities case and no effect when considering income for comfort.

5. DISCUSSION

5.1 Sensitivity of the Results

5.1.1 The Base Pension in the Discounted Utility Models

The sensitivity tests support the view that under the discounted utility framework, for a given base income, the rankings amongst the strategies are relatively stable against variations in the key parameters. However, varying the base income level can have a significant impact on the rankings.

This suggests the most important consideration in implementing the discounted utility model is an understanding of how pensioners measure the relative value of different levels of income and the starting level of assets relative to their income requirement.

5.1.2 The Bequest Motive and Other Assets

The DUL result was relatively insensitive to the bequest motive. This contradicts the findings of Blake, Cairns & Dowd (2003) who suggest increasing equity exposure as the bequest motive strengthens. However, the preferred strategy under the DUL model was a level annuity which does not allow for increased equity exposure while the preferred strategy under Blake, Cairns & Dowd (2003) was an income drawdown account.

Assets outside the fund, including assets that could be bequeathed, similarly had no impact.

5.1.3 Sensitivity to Relative Risk Aversion and the Utility Function

The rankings of the various strategies given a base pension of the level life annuity changed substantially for very low RRA parameter values which represent risk-seeking behaviour. However, the preferred strategy was relatively stable for moderate and high RRA parameter values. Discounted utility models are often criticised for being difficult to parameterise, however the results from this research suggest that unless the investor is very risk-seeking, the preferred annuitisation decision will not be overly sensitive to this parameter.

5.1.4 Sensitivity to Life Expectancy at Retirement

The scenarios using a female pensioner and using an earlier retirement age implicitly tested for greater longevity in retirement. In cases where expected longevity at retirement was longer, the ruin theory model avoided annuities because of their increased costs. The discounted utility model using an income for necessities level as the base case reduced the equity exposure if the income drawdown was higher and switched from a living annuity to a living annuity with annuitisation if funding levels were higher. The reduction in equity exposure is contrary to the preferences found in Milevsky & Robinson (2000). However, this finding was not unexpected given that discounted utility models give higher rankings to strategies that avoid the possibility of very low income levels while ruin theory models give higher rankings to riskier strategies with at least some prospect of higher income levels.

5.1.5 Sensitivity to Initial Funding Level

The ruin theory model was extremely sensitive to the initial funding level, or the drawdown rate relative to the initial wealth at retirement relative to the desired income level. If there was sufficient capital to purchase an inflation-linked annuity equal to the desired initial income or higher, the ruin theory model gave the inflation-linked life annuity the highest ranking. Otherwise a living annuity with relatively high equity allocation was selected. This is because in an underfunded position the ruin theory model will give higher rankings to strategies that have at least some possibility of not ending in ruin, even though the income threshold may be high. Because of the higher expected return on equities and greater volatility of equity returns, the probability of ruin is lower for higher equity exposure despite the higher probability of completely exhausting available funds.

The DUL model gave the highest ranking to the level annuity, irrespective of the funding level. Similarly, the DUI gave the highest ranking to fixed 5% escalation life annuities.

The DUN model was sensitive to the funding level but gave considerably more variation in the rankings. It typically favoured living annuities. Living annuities without the protection of later purchase of a life annuity were favoured if the funding level was high. Otherwise, living annuities with annuitisation later were ranked highest.

5.2 Different Results under Different Models

If there is a fixed investment term that is known in advance and insurance is ignored, Browne (1995) has shown that the optimal investment strategy under ruin theory and under discounted utility with an exponential utility function will be the same. If the future lifetime is not known and discounted utility is used with constant RRA, then the best investment mix for income drawdown accounts will be similar but not identical (Bayraktar & Young, 2007). However, once life annuities are considered, there is no prior research to suggest that the annuitisation decision should be similar under ruin theory and discounted utility.

The results of this research show that under most circumstances, the ruin theory model suggested different strategies than what discounted utility models would suggest. Even when the discounted utility models would also suggest living annuities, the ruin theory model ranked more aggressive investment strategies more highly than the discounted utility models. In addition, the ranking of various strategies under discounted utility models depended on the income level used to anchor the utility function.

5.2.1 Ruin Theory Rankings

It was noted that the results suggest that ruin theory models tend to select aggressive strategies, particularly for lives that have higher income needs relative to their savings. This was consistent with the literature as discussed in Section 2.4.1. The bequest motive was not explicitly included in our ruin theory model; however, the literature would suggest that the more under-funded a retiree would be relative to their income or bequest needs, the higher the equity exposure that would be suggested. Some commentators might view this as gambling.

Given the propensity of the ruin theory model to select aggressive strategies, it is noteworthy that even under the highest ranked strategy, the probability of ruin often remain significant. In addition, the extent of ruin is not clearly shown. It could be argued that a retiree may be uncomfortable with these probabilities in absolute terms, and may instead be willing to accept some level of shortfall relative to required levels upfront rather than run the risk of sharply poor investment performance jeopardising the entire living annuity strategy.

5.2.2 DUL Rankings

In contrast to the ruin theory criterion, the DUL rankings amongst the 11 strategies remained surprisingly stable to changes in demographics. Inflation risk was assumed in exchange for higher initial incomes.

This result supports the theory that most people would rather revise their goals downwards, than run the risk of falling materially short of these standards. In addition most people need high income levels relative to their savings at, and throughout, retirement due to the lack of adequate contributions in the lead-up to retirement. This is one possible reason why level annuities, which provide income over life, and provides the highest level of income out of the life annuities, are popular. A possible second driver of this effect is the interaction of the discount rate, implied discount factors and the ratio of tested strategy income to base pension in the utility function formulation. For example, an inflation-linked annuity defers the distribution of income relative to a level annuity, creating a strongly increasing ratio over time. The trend in the ratio varies by the strategy being tested and the benchmark or denominator used. This can then be amplified or dampened depending on the discount rate. Additionally, the formulation of the utility function uses a ratio raised to an exponent. Hence the utility result is highly sensitive to the ratio of actual income to the base level of income and the effect of the exponent on the ratio.

5.2.3 DUI Rankings

By changing the reference point from income under a level to an inflation-linked annuity, the most preferred strategy was a fixed 5% escalation annuity. Under the long-term assumption that inflation would remain below 6%, the upper band of the South African Reserve Bank's target, a 5% increase was expected to beat inflation in many simulations. This annuity also achieves some balance between relatively good future increases, and an upfront income that is not too low due to forgoing full inflation-proofing. Further research is required on the sensitivity of this result to the parameterisation of the asset model with regard to expected inflation.

5.2.4 DUN Rankings

When income for comfort was the threshold, aggressive lifestage annuities were favoured. When this threshold was changed to necessities, straight living annuities without annuitisation were favoured. The annuitisation aspect is consistent with discounted utility frameworks in general, which prioritise the management of longevity risk. Due to the need to keep up with inflation each year, aggressive asset allocations were favoured to achieve this.

5.3 Relative Advantages and Disadvantages of Models

There are advantages and disadvantages to both the ruin theory and discounted utility approaches as discussed below.

5.3.1 Ruin Theory

At face value, the probability of running out of funds is easy to understand and to relate to, and the model output can be used directly. However, the modelling behind ruin theory can be extremely complex and subjective and this is difficult to explain. In addition, in a country with low numeracy, comparing probabilities may be quite difficult for certain groups of fund members.

If there is insufficient capital to buy an inflation-linked annuity, ruin theory favours aggressive living annuities. This is because it does not measure depth of shortfall when a shortfall is experienced. This level of risk-seeking may be inconsistent with how all individuals would see this risk/reward trade-off.

Ruin theory is also very sensitive to starting capital, which means if full and accurate information is not available at the outset, it could produce a very different result to what it would if full information were available. It also means it is difficult to use with projected investment values. In addition, ruin theory requires fairly accurate forecasting of needs in retirement. A difference of a few rands in forecasted needs could change the outcome.

5.3.2 Discounted Utility

In contrast, discounted utility provides stability of the preferred solution across different demographic profiles and generally across different parameters, unless the annuitant is very risk-seeking. The results tend to favour strategies providing more stability around achieving the base outcome, as extent of shortfalls, when they occur, are taken into account in the analyses. In addition, to the authors, the results from the discounted utility model seemed more compatible with human behaviour.

On the other hand, the results are less intuitive and less easy to explain. While an individual may understand ranking of preferences, a utility level may not be meaningful. This complicates the comparisons across strategies. In addition, terms like 'risk-seeking' and 'risk-averse' may be difficult to explain. Although the elicitation of the utility functions may be challenging, Thomson (2003b) showed that discounted utility can be implemented in the context of a South African defined contribution fund.

It is important to note that the base pension chosen in the consumption utility function can affect preferred solution, for example:

— If income from level annuities were used, level annuities were the best option.
— If income from inflation-linked annuities were used, fixed-escalation at 5% or inflation-linked annuities were preferred, but a conservative living annuity with annuitisation solution came up as a reasonable alternative.
— If income for necessity levels were used in the denominator, moderate to aggressive living annuities with annuitisation were favoured especially when income for comfort is required to be met.

5.4 Areas for Future Research

This paper is limited to considerations around the annuitisation decision. However, it is noted that the proportion of retirement wealth to take as a cash lump sum at retirement is non-trivial and requires further investigation.

This research considered purchasing inflation-linked annuities after holding a living annuity. Given current annuity prices, fixed-increase and with-profits annuities would be more affordable and these options should be included in future research.

Further research is required into appropriate ages for holders of living annuities to purchase life annuities.

The extension of the work to higher income earners, liable for income tax, is regarded as a fairly important extension. However, it is expected that there will be little change to the preferences under ruin theory, and given the progressivity of the South African taxation regime, the aggressive living annuities may appear less preferable under the discounted utility models.

More work is also required on the formulation of the utility function, its parameterisation and its arguments, particularly with regard to whether income should be measured relative to a base strategy or in absolute terms. The treatment of reversionary annuity benefits in discounted utility models requires further consideration

Further research is required as to:

— Preferred strategies given health shocks in retirement;
— The impact of other demographic factors such as education and income levels on the results; and
— The sensitivity of the results obtained to the asset model used.

6. IMPLICATIONS AND CONCLUSIONS

Although Milevsky & Robinson (2000) proposed their model as a way of helping individuals to make decisions regarding income drawdown accounts and in particular the degree of equity exposure in the income drawdown accounts, this research indicates that the preferred annuitisation strategy under one model may be very different from that under another equally plausible model.

The chief difficulties are that each fund may not have access to all of the individual's financial information and individuals may not to know their own information either because their investments are subject to market fluctuations or because they have lost track of their finances. This could make automated suggestions or member-populated models very misleading.

Given the sensitivity of the result to funding levels and retirement budgets, separating the annuitisation decision from general financial coaching may produce sub-optimal results. In addition, it would be difficult to find a "one-size-fits-all" solution that is appropriate for everyone or to suggest a preferred annuitisation strategy given the information available to the trustees of the retirement fund.

Consequently, it would be risky for trustees to put members into annuity products without advice. Trustees may not wish to take on this responsibility without legislative protection. If trustees leave the annuitisation decision with members and make a discounted utility model available to members to guide them in their decision, Thomson (2003a) suggests that trustees may either require that the axioms be explained to members or a suitable warning be used explaining that the results may be inconsistent with the individual's approach to preferences. On-site education and training may also be required to facilitate the process (Thomson, 2003b).

The living annuity can form part of a preferred annuitisation strategy under all of the models and this suggests that it might be inappropriate to prevent people from using living annuities altogether.

In conclusion, all the discounted utility methods tended to reward certainty which seems to reflect actual behaviour and hence may appear palatable to members as well as being consistent with the goals of National Treasury. However, the discounted utility models produce results that are very different from those produced using ruin theory, another arguably valid framework. This suggests that using a single model without considerations of the advantages, disadvantages and characteristics of the model may result in inappropriate annuitisation decisions.

ACKNOWLEDGMENTS
The authors express their gratitude to Sebastian Commin for producing many of the inputs used in the modelling done for this paper. Thanks are also due to the various members of the Research and Product Development team at Alexander Forbes who provided comments and insights. Special thanks are due to: Anne Cabot-Alletzhauser, John Anderson, Youri Dolya and Sandira Chaithram.

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