ACTUARIAL SOCIETY OF SOUTH AFRICA

How a single-factor CAPM works in a multi-currency world

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"A grand adventure is about to begin"

-Winnie-the-Pooh

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AGENDA

1. Background
2. Aim
3. Necessary and sufficient condition
4. Single-factor multi-currency CAPM (SFM-CAPM)
5. Maximum likelihood estimation of betas
6. Ordinary least squares estimation of betas
7. Data
8. Results
9. Further research

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So why did we decide to write this paper?

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Showed that there is no unique single-factor CAPM in a multi-currency world.

…the standard CAPM cannot describe the pricing of capital assets for both investors.

The standard CAPM assumes that all investors measure risk and return in the same currency.

"Why the capital-asset pricing model fails in a multi-currency world" by A.D. Wilkie

Showed that if two investors measure risk and return in different currencies…

Therefore when different currencies exist…

…the standard CAPM's assumption that all investors measure risk in the same currency breaks down.

Let's assume that different domestic capital markets are independent entities…

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Segregated market

- A market in which the price of an asset depends on where it is traded (Karolyi & Stulz, 2003).
- A domestic CAPM is appropriate only for an asset that is traded in a closed, domestic financial market (Stehle, 1977; Stulz, 1995a).

Domestic CAPM

$$E\{R_i\} = R_F + \beta_i[E\{R_M\} - R_F]$$

…then the international market consists of individual segregated markets (Solnik, 1974b).

But is this the case in reality?

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Integrated market

- A market in which assets have the same price regardless of where they are traded (Karolyi & Stulz, 2003).
- Limitations to use of domestic CAPM
- Home bias
- Development of equilibrium models

No, because we have access to international markets.

Hence, the international CAPM (ICAPM)…

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ICAPM

- Purchasing power parity does not hold perfectly.
- Exchange-rate risk is priced by modifying the CAPM.
- Only consider the single-factor and multi-factor ICAPM.
- Both look at the world from the perspective of one currency only.

…and different versions thereof were developed.

The single-factor ICAPM or global CAPM (GCAPM)…

$$E\{R_i\} = R_F + \beta_i^w [E\{R_W\} - R_F]$$

…looks at the world from the perspective of one currency only–the currency in which the investor measures returns.

If strict purchasing power parity applies

AND

If returns are measured in real terms (or there is no inflation)

THEN

The GCAPM applies

THEREFORE

Risk-free rate

Mean-variance optimal portfolio

Variances and covariances of return and beta

Equal

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In the light of Wilkie's conclusion and the literature reviewed …

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How a single-factor CAPM works in a multi-currency world

- Single-factor CAPM in a multi-currency world
- Unique CAPM

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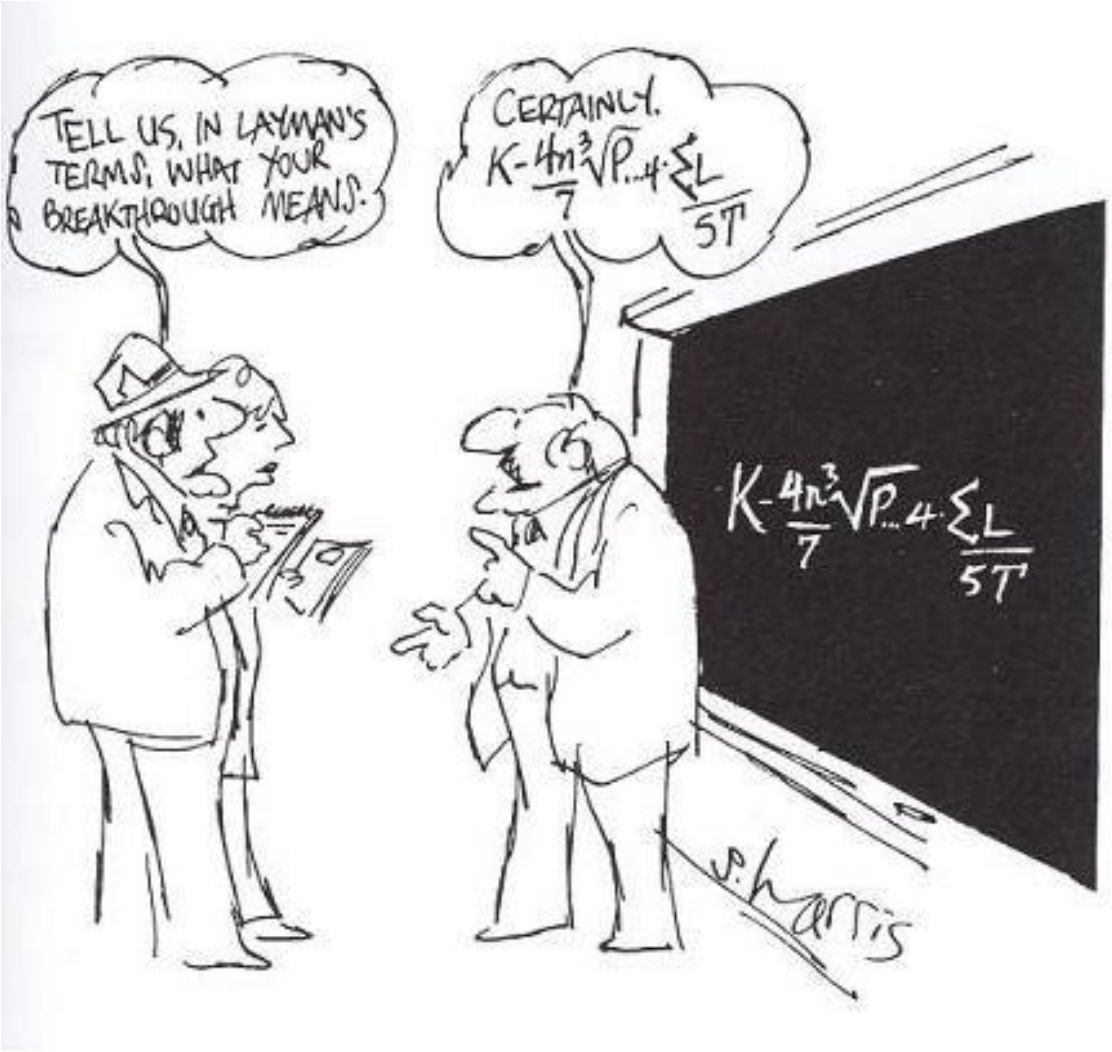
But I will try…



A preliminary discussion is required

- *C* currencies
- In currency *c*, there is one risk-free asset and n_c risky capital assets.
- An ‘asset issued in currency *c*’ is a risky asset issued in that currency or the risk-free asset denominated in that currency.
- ‘currency-c’ investor
- Forces of return over a unit interval.

We made further assumptions based on the CAPM

- We assume that the CAPM applies for investors in each currency. More specifically, we assume that:
  - ‘currency-*c* investors’ have indifference curves in mean–variance space
  - investors have homogeneous expectations

It is shown that for a single-factor CAPM to work in a multi-currency world, there is a necessary and sufficient condition…

$$\sigma^{c}_{di,\mathrm{M}} + \sigma^{c}_{c,\mathrm{M}} = a_c \sigma^{1}_{di,\mathrm{M}} \quad \text{where} \quad a_c = \frac{\sigma^{c}_{\mathrm{M,M}}\left(\mu^{1}_{\mathrm{M}} - x_{10}\right)}{\sigma^{1}_{\mathrm{M,M}}\left(\mu^{c}_{\mathrm{M}} - x_{c0}\right)}$$

holds if and only if

$$\mu^{c}_{di} = x_{c0} + \frac{\sigma^{c}_{di,\mathrm{M}}}{\sigma^{c}_{\mathrm{M,M}}}\left(\mu^{c}_{\mathrm{M}} - x_{c0}\right)$$

holds

…that applies to the *ex-ante* variances and covariances of return.

Formulation of the SFM-CAPM

$$\sigma_{di,\mathrm{M}}^{c} + \sigma_{c,\mathrm{M}}^{c} = a_c \sigma_{di,\mathrm{M}}^{1} \quad \text{where} \quad a_c = \frac{\sigma_{\mathrm{M,M}}^{c}\left(\mu_{\mathrm{M}}^{1} - x_{10}\right)}{\sigma_{\mathrm{M,M}}^{1}\left(\mu_{\mathrm{M}}^{c} - x_{c0}\right)}$$

may also be expressed as:

$$\left(\sigma_{di,\mathrm{M}}^{c} + \sigma_{c,\mathrm{M}}^{c}\right)\frac{\mu_{\mathrm{M}}^{c} - x_{c0}}{\sigma_{\mathrm{M,M}}^{c}} = \boxed{\sigma_{di,\mathrm{M}}^{1}\frac{\mu_{\mathrm{M}}^{1} - x_{10}}{\sigma_{\mathrm{M,M}}^{1}}}$$

In other words, for all c:

$$\left(\sigma_{di,\mathrm{M}}^{c} + \sigma_{c,\mathrm{M}}^{c}\right)\frac{\mu_{\mathrm{M}}^{c} - x_{c0}}{\sigma_{\mathrm{M,M}}^{c}} = \boxed{\kappa_{di}} \Rightarrow \boxed{\text{Adjusted risk premium}}$$

This specifies the single factor for the SFM-CAPM

From

$$\left(\sigma^{c}_{di,\mathrm{M}} + \sigma^{c}_{c,\mathrm{M}}\right)\frac{\mu^{c}_{\mathrm{M}} - x_{c0}}{\sigma^{c}_{\mathrm{M,M}}} = \kappa_{di}.$$

$$\beta^{c}_{di} = \frac{\sigma^{c}_{di,\mathrm{M}}}{\sigma^{c}_{\mathrm{M,M}}} \qquad \beta^{c}_{c} = \frac{\sigma^{c}_{c,\mathrm{M}}}{\sigma^{c}_{\mathrm{M,M}}}$$

zero

Therefore this may be expressed as:

linear constraint

$$\beta^{c}_{di,c}\left(\mu^{c}_{\mathrm{M}} - x_{c0}\right) = \kappa_{di}$$

where $\beta^{c}_{di,c} = \beta^{c}_{di} + \beta^{c}_{c}$

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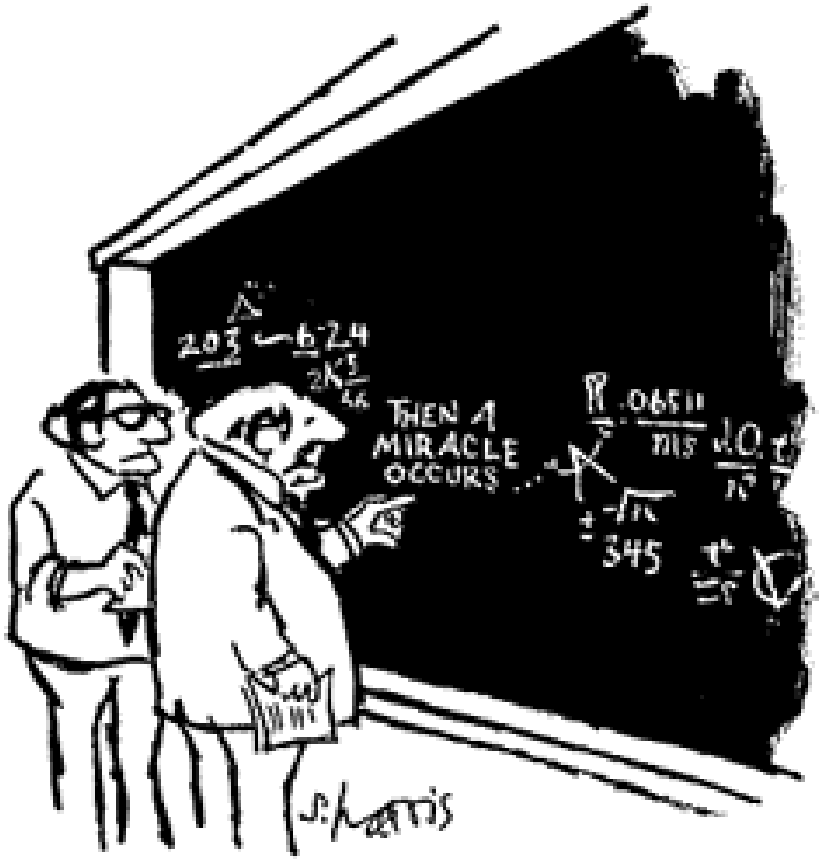
The derivation of the SFM-CAPM and the necessary and sufficient condition will not be discussed…



"I think you should be more explicit here in step two."

…however the SFM-CAPM developed in this paper may be specified as:

$$E\left\{X_{di}^{c}\right\} = x_{c0} + \beta_{di,c}^{c}\left(E\left\{X_{\mathrm{M}}^{c}\right\} - x_{c0}\right)$$

is the return in currency *c* on asset *i* issued in currency *d*

is the return in the risk/unbiased currency *c* on the asset denominated in currency *d*

is the risk/unbiased beta plus the additional beta

is the return in currency *c* on the optimal market portfolio of a currency-*c* investor

$c, d = 1, \ldots, C;\ i = 0, 1, \ldots, n_d$

If strict purchasing-power parity holds then the SFM-CAPM reduces to the GCAPM since the additional beta term will be zero.

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How do we estimate the covariance and variances of return in the SFM-CAPM using the constraint?

$$E\left\{X_{di}^{c}\right\} = x_{d0}^{c} + \boxed{\beta_{di,c}^{c}}\left(\mu_{\mathrm{M}}^{c} - x_{c0}\right)$$

$$\boxed{\beta_{di,c}^{c}} = \boxed{\beta_{di}^{c}} + \boxed{\beta_{c}^{c}}$$

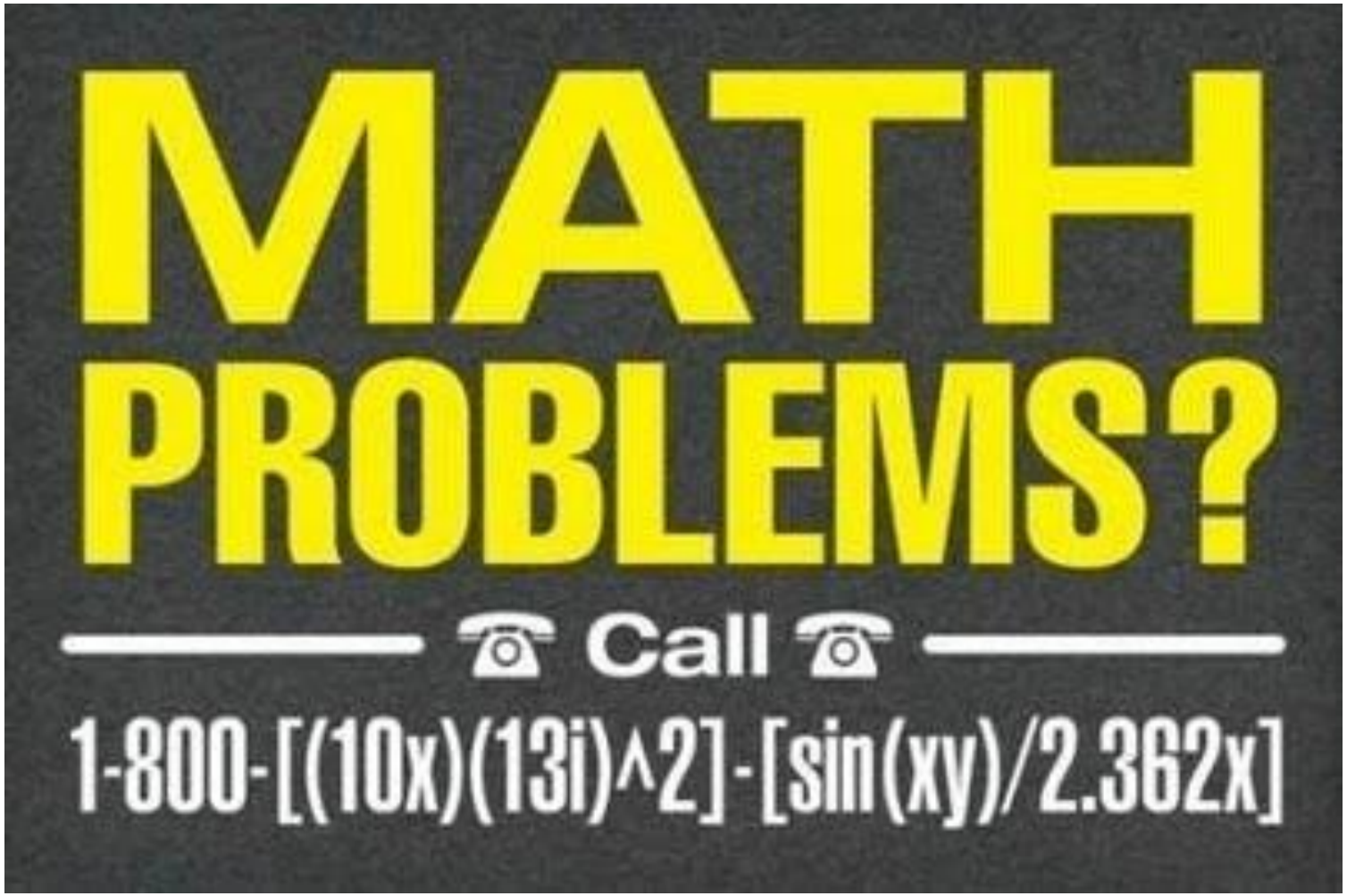
$$\boxed{\beta_{di}^{c}} = \frac{\boxed{\sigma_{di,\mathrm{M}}^{c}}}{\boxed{\sigma_{\mathrm{M,M}}^{c}}} \qquad \boxed{\beta_{c}^{c}} = \frac{\boxed{\sigma_{c,\mathrm{M}}^{c}}}{\boxed{\sigma_{\mathrm{M,M}}^{c}}}$$

$$\boxed{\sigma_{di,ej}^{c} \;\&\; p_{di}^{c}}$$

$$\boxed{\sigma_{di,\mathrm{M}}^{c} + \sigma_{c,\mathrm{M}}^{c} = a_{c}\sigma_{di,\mathrm{M}}^{1}}$$

constrained maximum likelihood estimation (MLE)

But there are problems with the MLE approach



While the MLE approach is theoretically appealing…

- Complicated
- Lengthy program
- Iterative process which may fail to converge
- Inversion of a large matrix
- Singular matrices
- Short positions
- Constrained variance not necessarily positive
- Correlation coefficients out of [-1,1] range

…it is problematic to apply in practice.

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An alternative approach, using ordinary least squares, is developed.

$$E\left\{X_{di}^{c}\right\} = x_{d0}^{c} + \boxed{\beta_{di,c}^{c}}\left(\mu_{\mathrm{M}}^{c} - x_{c0}\right)$$

$$\boxed{\beta_{di,c}^{c}\left(\mu_{\mathrm{M}}^{c} - x_{c0}\right) = \kappa_{di}}$$

$$\boxed{\frac{\partial}{\partial \kappa_{di}}\left[\sum_{c=1}^{C}\left\{\beta_{di,c}^{c}\left(\mu_{\mathrm{M}}^{c} - x_{c0}\right) - \kappa_{di}\right\}^{2}\right] = 0}$$

$$\boxed{\hat{\kappa}_{di} = \frac{1}{C}\sum_{c=1}^{C}\left\{\boxed{\hat{\beta}_{di,c}^{c}}\left(\hat{\mu}_{\mathrm{M}}^{c} - x_{c0}\right)\right\} = \frac{1}{C}\sum_{c=1}^{C}\hat{\kappa}_{di}^{c}}$$

$$\beta_{di,c}^{c} = \beta_{di}^{c} + \beta_{c}^{c}$$

"original" beta

$$\boxed{\hat{\beta}_{di,c}^{c}} = \frac{\boxed{\hat{\kappa}_{di}}}{\hat{\mu}_{\mathrm{M}}^{c} - x_{c0}}$$

Least-squares estimates of beta

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The theory is applied to two major currencies and two minor currencies…

…and the following periods and assets were used.

- Selection of periods from 1975Q2 to 2012Q1 for nominal returns and from 2003Q2 to 2012Q3 for real returns.
- Risky assets:
  - Comprehensive index of equities
  - Two conventional bonds: short (one-quarter) bond and a long-term bond
  - Two index-linked bonds: short (one-quarter) bond and a long-term bond
- Risk-free asset:
  - Nominal returns: short conventional bond
  - Real returns: short index-linked bond
- "How a single-factor CAPM works in a multi-currency world: information on the determination of data"

For example

Data set	Period		USA		UK		SA		TR	
			e, cb	ilb	e, cb	ilb	e, cb	ilb	e, cb	ilb
nominal returns										
1	1975Q2	1985Q4	√		√		√			
2	1986Q1	1995Q4	√		√		√		√	
3	1996Q1	2005Q2	√		√		√		√	
4	2005Q3	2012Q1	√		√		√		√	
5	2005Q3	2012Q1	√	√	√	√	√	√		
6	2009Q4	2012Q1	√	√					√	√
7	1975Q2	2012Q1	√		√		√			
8	1986Q1	2012Q1	√		√		√		√	
real returns										
1	2003Q2	2009Q3	√	√	√	√				
2	2005Q3	2009Q3	√	√	√	√	√	√		
3	2009Q4	2012Q1	√	√					√	√
4	2005Q3	2012Q3	√	√	√	√	√	√		
5	2003Q2	2012Q3	√	√	√	√				

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What did we compare?

$$\beta_{di,c}^{c} = \beta_{di}^{c} + \beta_{c}^{c}$$

$$\beta_{di}^{c} = \frac{\sigma_{di,\mathrm{M}}^{c}}{\sigma_{\mathrm{M,M}}^{c}}$$

unbiased sample betas

$$\hat{\beta}_{di,c}^{c} = \frac{\hat{\kappa}_{di}}{\hat{\mu}_{\mathrm{M}}^{c} - \hat{\mu}_{c0}}$$

least-squares estimates of the betas

$$\beta_{di,c}^{c} = \beta_{di}^{c} + \beta_{c}^{c}$$

“original” betas

For nominal and real returns for each period.

What do we expect?

Least-squares estimates would be approximately equal to the sample betas

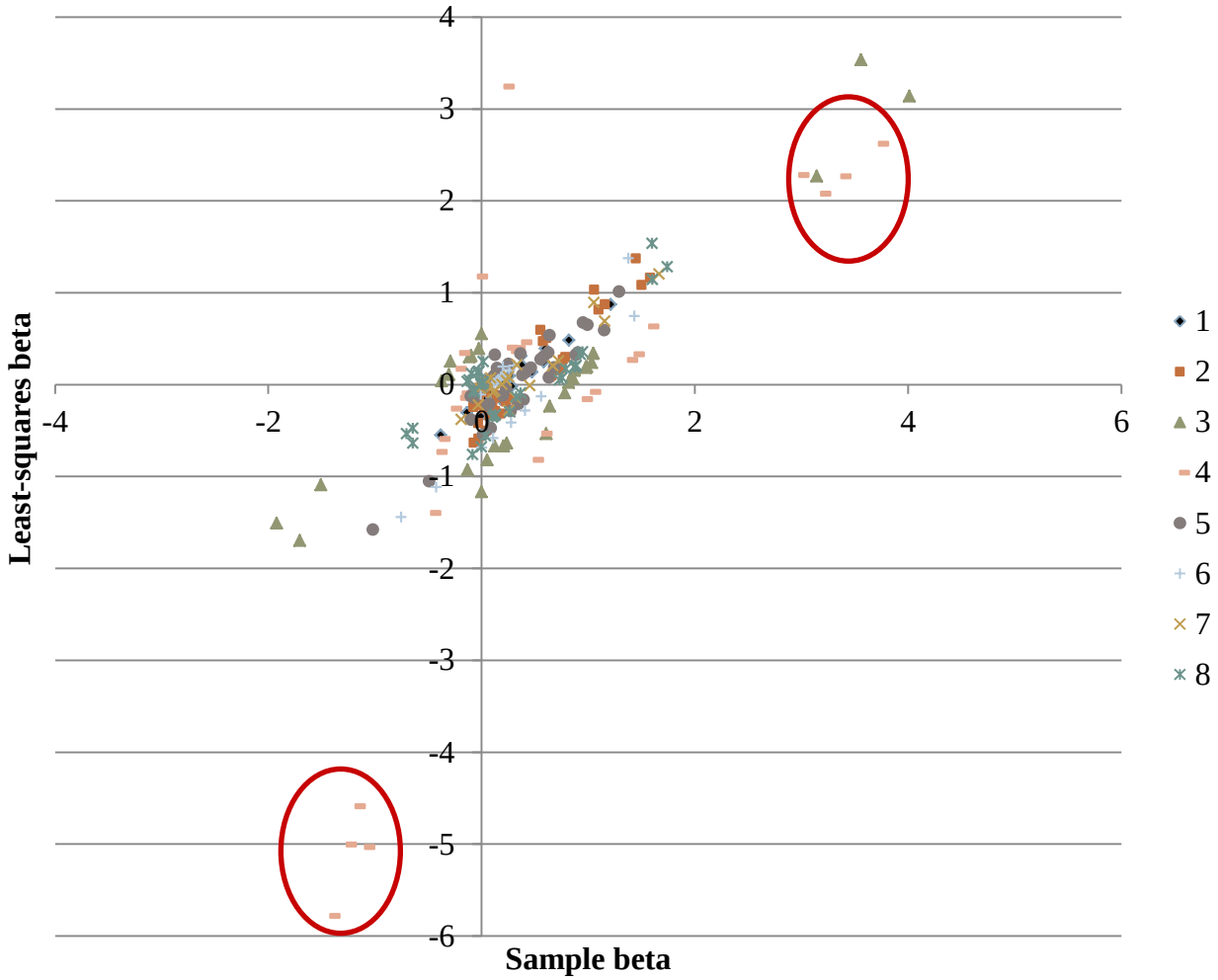
BUT

with some shift to represent effects of the constraint, additional beta and some noise

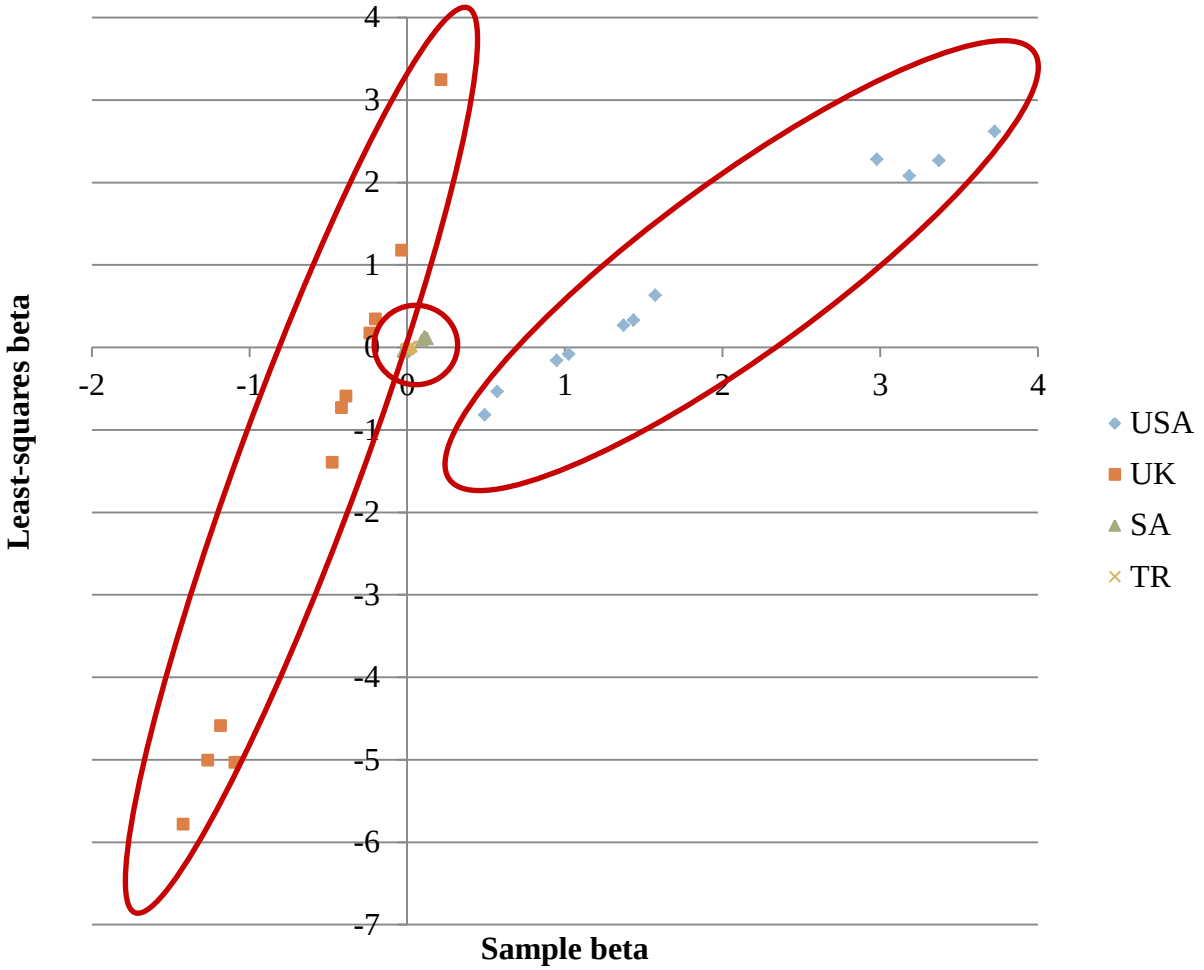
Sample betas are understated

Sample betas are overstated

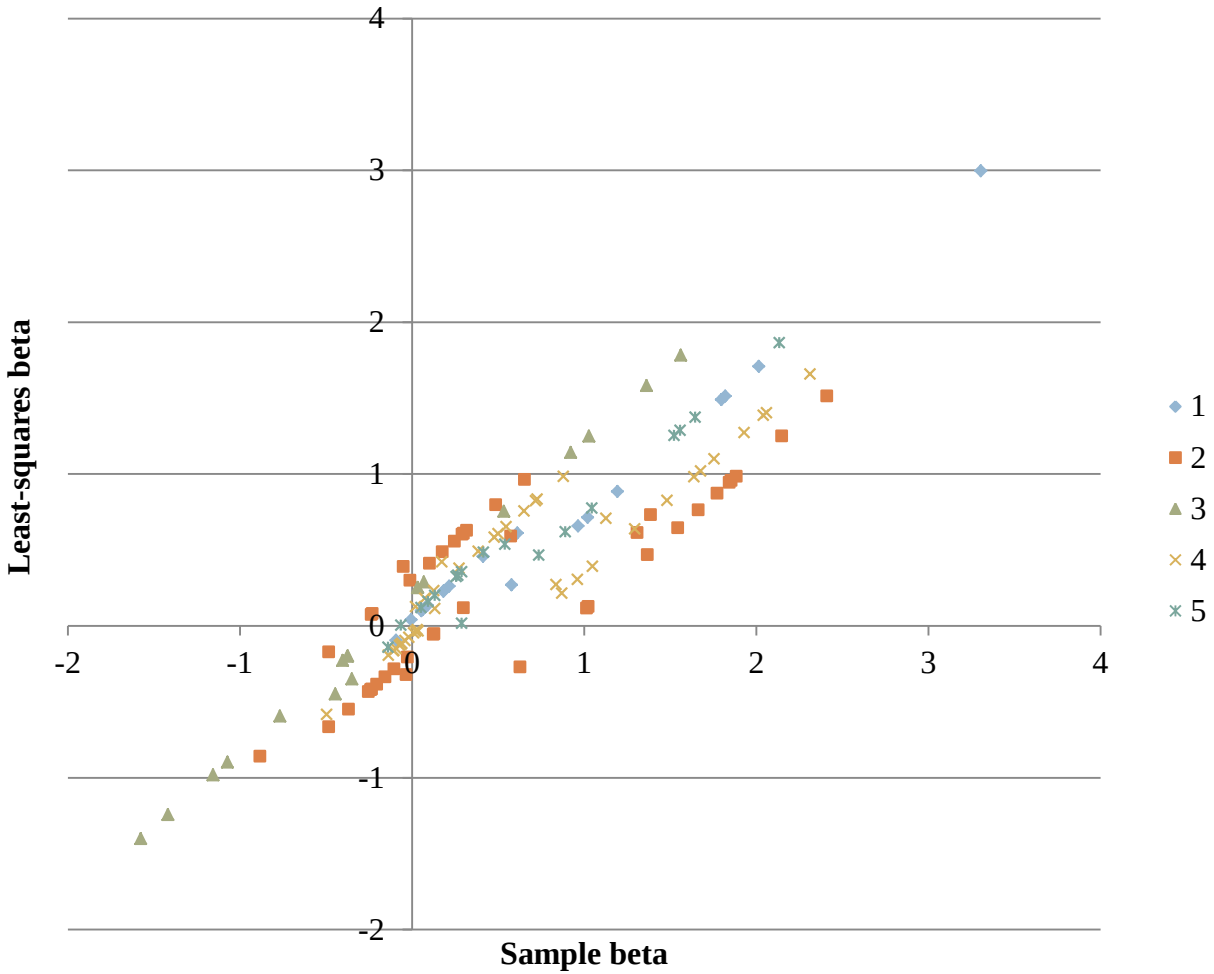
Least-squares beta versus sample beta: nominal returns



Let's look at dataset 4 a bit closer



Least-squares beta versus sample beta: real returns



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Findings of this paper will lead to further research…

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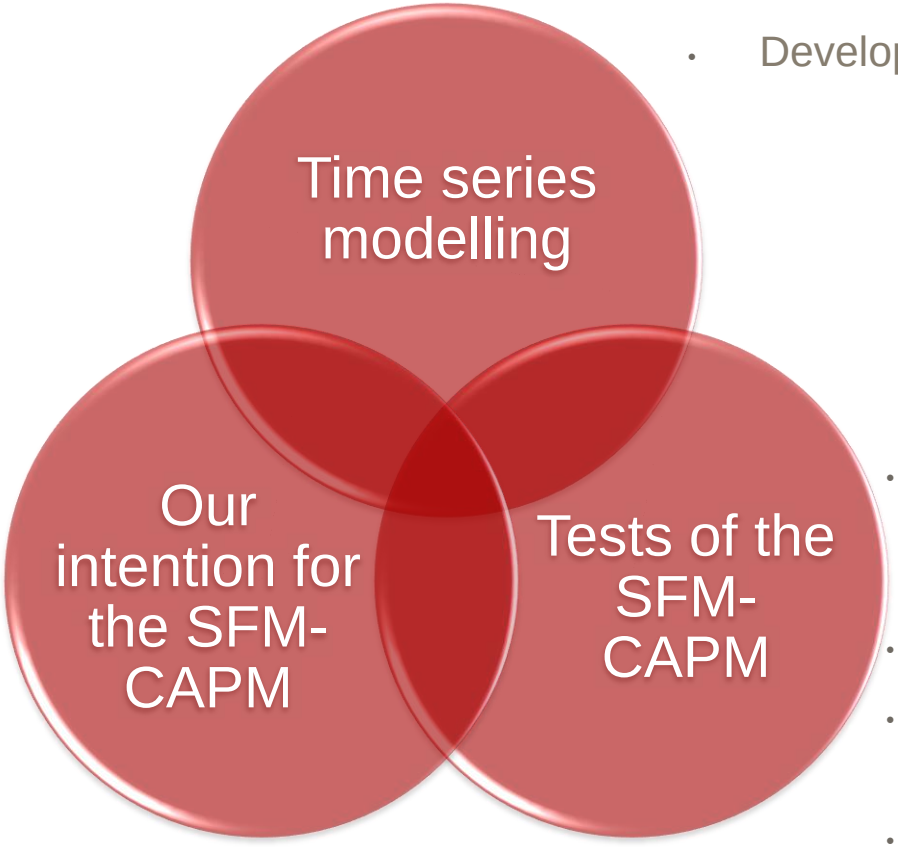
- Develop equilibrium models



- Take its place alongside other models in informing subjective assessment

- Ex-ante expected returns versus ex-post estimations
- Short positions
- Difficulties with MLE approach
- Recommend LSE method

Teşekkürler
Thank you